Instability in a laser-diode-pumped microchip Nd:YAG laser in a II scheme

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Dynamical instability, chaotic pulsations, and generalized bistability have been observed in a laser-diode-pumped microchip Nd:YAG laser operating in a double transition scheme in which lasing occurs on two transitions with overlapping gain profiles, \(4F_{3/2}(2) \rightarrow 1I_{11/2}(3)\) and \(4F_{3/2}(1) \rightarrow 1I_{11/2}(2)\), and simultaneously involves excited Nd atoms from different sublevels of the upper manifold. The modeling of the experimental results requires rate equations that include cross-gain coupling among oscillating modes that belong to different transitions whose population inversion densities are determined by the Boltzmann distribution. © 2001 Optical Society of America

It is known that single-mode class-B lasers, in which polarization dynamics is adiabatically eliminated, exhibit no dynamical instabilities without introducing additional degrees of freedom in the form of modulations, saturable absorbers, intracavity second-harmonic generation, light injections, external feedback, and so on. Recently, several self-induced instabilities that result from the inherent nonlinearities of multimode class-B lasers were demonstrated.\(^1\)\(^-\)\(^6\) In microchip LiNdP\(_2\)O\(_4\)/\(2\) lasers, atomic interference arises as a result of inherent simultaneous oscillations on different transitions that form a \(\Lambda\) scheme. This nonlinear effect becomes apparent because the photon lifetime is an order of magnitude shorter than the lifetime of lower-level populations in a short-cavity configuration. Low-frequency coherence results in intensity pulsations.\(^1\)\(^-\)\(^3\) As for Nd:YVO\(_4\) solid-state lasers, because of the substantially large inhomogeneous broadening that results from unusual crystal fields, intensity-dependent cross-gain coupling among longitudinal modes induces such dynamical instabilities as locking of pulsation frequencies, quasi-periodic and chaotic oscillations, and \(Q\)-switching-type spiking oscillations and associated instabilities.\(^4\)\(^-\)\(^6\) In this Letter we demonstrate, for the first time to our knowledge, self-induced pulsations in a laser-diode-pumped Nd:YAG laser in the so-called II scheme, in which lasing occurs that involves 1064.1- and 1064.5-nm transition oscillations. The key behind the observed instability is that lasing takes place that involves excited Nd atoms in Stark-split upper laser levels whose population distribution obeys Boltzmann’s law, whereas the Nd populations in the lower laser levels are negligibly small. These properties induce coupling between two otherwise isolated two-level transitions.

The experiment was carried out with a 1% Nd-doped YAG crystal with a 5-mm thickness (Casix, DPO7104) in which dielectric mirrors (transmission, 0.2% and 5% for each end surface) were coated. A collimated laser-diode pump beam was focused on the crystal by a microscope objective lens (magnification, 20×; numerical aperture, 0.40) after it passed through a pair of anamorphic prisms. With such circular-shaped beam pumping, TEM\(_{00}\) mode oscillations were obtained. We used a monochromator with a resolution of 0.1 nm to select the modal output. The oscillation spectrum was measured with a high-resolution scanning Fabry–Perot interferometer (Newport SR-250-C; resolution, 0.26 GHz). The modal output waveform and power spectrum were measured with a low-noise photoreceiver (New Focus 1811; dc bandwidth, 125 MHz) followed by a digital oscilloscope (Tektronix 420B; sampling rate, as much as 100 MHz; bandwidth, 500 MHz) and a rf spectrum analyzer (HP8591A; bandwidth, 9 kHz–1.8 GHz).

When the pump power was increased from zero, the first longitudinal lasing mode, \(s_1\), appeared at 1064.1 nm, which corresponds to the \(4F_{3/2}(2) \rightarrow 4I_{11/2}(3)\) (\(l_1\) line) transition whose fluorescence linewidth is 0.62 nm. This is because the 1064.1-nm line has the largest effective emission cross section.\(^7\) The threshold pump power was \(P_{th} = 170\) mW, and a single TEM\(_{00}\) mode oscillation was obtained. At pump power \(w = P/\lambda P_{th} = 1.2\), a second longitudinal lasing mode, \(s_2\), appeared at 1064.5 nm. This mode belongs to the \(4F_{3/2}(1) \rightarrow 4I_{11/2}(2)\) (\(l_2\) line) transition with a linewidth of 0.46 nm and an effective emission cross section 0.62 times that of the 1064.1-nm line,\(^8\) although the longitudinal-mode separation is calculated to be \(\Delta \lambda = \lambda^2/2nL = 0.062\) nm (\(\lambda\) is the lasing wavelength; \(n = 1.82\) is the refractive index;
$L = 5 \text{ mm}$ is the crystal length). This sparse-mode oscillation of the transitions separated by $6\Delta \lambda$ can be interpreted as follows: As the 1064.1-nm oscillation occurs, the upper laser-level population is frozen to the threshold value. Hence in the usual situation the longitudinal mode that belongs to other transitions whose emission cross sections are smaller than that of the first lasing mode transition should not begin to oscillate. In the II scheme, however, the 1064.5-nm transition mode can be the second lasing mode because the upper-level population density in the upper manifold is always 1.5 times greater than that of 1064.1-nm transition modes at room temperature as a result of the Boltzmann distribution. Longitudinal modes that belong to the homogeneously broadened 1064.1-nm transition line are globally coupled by cross saturation of population inversions through the spatial hole-burning effect, whereas they are decoupled with modes that belong to the 1064.5-nm transition. Consequently, the longitudinal mode that belongs to the 1064.5-nm transition reaches the lasing threshold earlier than other modes separated by $\Delta \lambda$ that belong to the 1064.1-nm transition because of its smaller cross-saturation effect. When the pump power was increased further in this two-transition–two-mode regime, periodic pulsation appeared first near $w = 2.25$ by means of a Hopf bifurcation followed by a wide period-2 pulsation region. Typical waveforms and the corresponding power spectra for the 1064.5-nm mode $s_2$ are shown in Figs. 1(a) and 1(b). The 1064.1-nm mode $s_1$ exhibited higher-frequency pulsations whose envelope is modulated at the pulsation frequency of the 1064.5-nm mode in Fig. 1. As the pump power was increased above $w = 3$, a third lasing mode, $s_3$, at 1063.9 nm that belongs to the $l_1$ transition appeared, and a narrow region of period-1 pulsation followed by chaos took place. Examples of period-1 and chaotic pulsations for the 1064.5-nm mode $s_2$ are shown in Figs. 1(c)–1(f).

When the pump power was decreased from the chaotic region to zero, we observed the reverse situation: chaos $\rightarrow$ period-1 $\rightarrow$ period-2 $\rightarrow$ period-1 $\rightarrow$ damped oscillations. Interestingly, however, the pulsation was not quenched near $w = 2.25$, the Hopf bifurcation point observed in the forward-pumping process, but survived down to $w = 1.9$. In addition, a qualitative change occurred in the periodic output near $w = 2.5$, where a transition from harmonic to spiking output was observed. Thus there is coexistence, i.e., bistability, between two classes of periodic output and bistability between periodic and steady regimes that is similar to what is observed in lasers with incoherent delayed feedback.\(^\text{10,11}\)

We theoretically elucidate the observed dynamics by modeling a II scheme whose population inversion distribution obeys Boltzmann’s law. We introduce cross saturation of population inversions among modes that belong to the same transition and direct cross-gain coupling among modes that belong to different transitions. The former coupling comes from the spatial hole-burning effect.\(^\text{9}\) The latter nonlinearity results from the overlap of the two homogeneously broadened fluorescence lines, located at 1064.1 and 1064.5 nm.

In short, the small-signal gain of one transition decreases because of an increase in the modal intensity of the other transition through the intensity-independent gain saturation. Such a coupling mechanism is similar to what arises in inhomogeneously broadened Nd:YVO$_4$ lasers.\(^\text{4–6}\) We thus propose for three-mode oscillations the model equations that are closest to the experimental situation (i.e., two modes on the $l_1$ line and one mode on the $l_2$ line) in the form

$$\frac{ds_1}{dt} = K[(n_0 - n_1/2)(1 - \eta s_2) - 1]s_1,  \quad (1)$$
$$\frac{ds_2}{dt} = K[g_2(\beta n_0 - n_2/2)(1 - \eta s_1 - \eta s_3) - 1]s_2,  \quad (2)$$
$$\frac{ds_3}{dt} = K[g_3(n_0 - n_3/2)(1 - \eta s_2) - 1]s_3,  \quad (3)$$
$$\frac{dn_0}{dt} = w - n_0(1 + s_1 + g_3s_3) + (n_1s_1 + g_3n_3s_3)/2,  \quad (4)$$
$$\frac{dn_1}{dt} = -n_1(1 + s_1 + g_3s_3) + n_0s_1,  \quad (5)$$
$$\frac{dn_2}{dt} = -n_2(1 + g_2s_2) + \beta g_2n_0s_2,  \quad (6)$$
$$\frac{dn_3}{dt} = -n_3(1 + s_1 + g_3s_3) + g_3n_0s_3,  \quad (7)$$

The normalized photon intensity is $s_i$ ($i = 1, 2, 3$). The first ($s_1$) and the third ($s_3$) lasing modes are on the $l_1$ line (1064.1 nm); the second mode, $s_2$, is on the $l_2$ line.

Fig. 1. Observed waveforms and corresponding power spectra of the second lasing mode, $s_2$. (a), (b) $w = 2.6$; (c), (d) $w = 3.3$; (e), (f) $w = 3.5$. (a) Period-2 pulsation was observed in pump region $w > 2.5$ and (c) period-1 pulsation appeared in a quite narrow pump region below chaotic oscillation.
(1064.5 nm). \( n_0 \) is the space average of the population inversion densities for modes operating on the \( l_1 \) line; \( n_{1,3} (n_2) \) are the first-order Fourier components of population inversion densities for \( s_{1,3} (s_2) \) on the \( l_1 \) line (\( l_2 \) line). All population densities are normalized by the first lasing mode's threshold intensity. \( K \) is the ratio of fluorescence lifetime to photon lifetime. \( K \) is the pump power normalized by the first lasing mode threshold, \( g_i (i = 2, 3) \) is the linear gain of mode \( s_i \), divided by the first lasing mode \( g_1 \). \( \eta \) is the cross-gain coupling coefficient, and \( t \) is the time scaled to the fluorescence lifetime.

In this scheme, the order of lasing modes is ruled not solely by the relative gains \( g_i \) as is usual but also by the fill factor \( \beta \). In the way Eqs. (1)–(7) are scaled, mode \( s_1 \) always lases first. Yet the order of the other lasing modes depends on the ratio \( r = \beta g_2 / g_3 \). If \( r > 1 \) \( (r < 1) \), mode 2 (3) is the second lasing mode. Exact expressions have been obtained for the steady state, whose linear stability has been assessed in the double limit of large \( K \) and small \( \eta \). For a set of parameters that correspond to the experimental conditions \( K = 1000, \eta = 0.002, \beta = 1.5, g_2 = 0.62, \) and \( g_3 = 0.7 \), mode \( s_1 \) starts to lase at \( w = 1 \), then mode \( s_2 \) begins to lase for \( w > w_{th,2} = 1.24 \), and, finally, a three-mode regime sets in for \( w > w_{th,3} = 2.94 \). For these parameters, \( s_1 \) and \( s_2 \) cross twice, once in the two-mode region and once in the three-mode region, but this has no influence on stability, which is determined by the real part of the eigenvalues \( \lambda_j \) of the linearized characteristic equation. All the Re(\( \lambda_j \), as functions of \( w \), are negative, except one whose sign is changed at \( w = w_H \approx 2.25 \). This means that the two-mode steady state is stable until \( w_H \), above which it is destabilized by a Hopf bifurcation, whereas the steady-state three-mode regime is entirely unstable, in agreement with the experiments. Simulation results are shown in Fig. 2 as a bifurcation diagram for the 1064.5-nm mode \( s_2 \), where peak values of intensity pulsation are plotted versus pump power, with the assumption of the same parameters as above: filled (open) circles correspond to forward (backward) pumping. The occurrence of a Hopf bifurcation and the numerical simulations based on Eqs. (1)–(7) are in excellent agreement with the experimental observations.

In summary, Hopf instability, chaotic pulsations, and generalized bistable behaviors that result from intensity-dependent cross-gain coupling between oscillating modes that belong to different transitions in a II scheme have been demonstrated in a laser-diode-pumped Nd:YAG laser. The key mechanism of instability is the simultaneous oscillation of transitions at 1064.1 and 1061.5 nm, in which excited Nd atoms in Stark-split upper laser levels whose populations obey the Boltzmann distribution are involved in lasing.

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