Chapter 1 An Outline of Generalized Linear Models

1.1 Introduction:

Linear models:

$$Y=\begin{bmatrix} Y_1 \\ Y_2 \\ \vdots \\ Y_n \end{bmatrix}, X=\begin{bmatrix} x_{11} & x_{12} & \cdots & x_{1p} \\ x_{21} & x_{22} & \cdots & x_{2p} \\ \vdots & \vdots & \ddots & \vdots \\ x_{n1} & x_{n2} & \cdots & x_{np} \end{bmatrix}, \varepsilon=\begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \vdots \\ \varepsilon_n \end{bmatrix}, \beta=\begin{bmatrix} \beta_1 \\ \beta_2 \\ \vdots \\ \beta_p \end{bmatrix}$$

and

$$Y = X\beta + \varepsilon,$$

with the following assumptions

$$\epsilon_i \sim N(0, \sigma^2)$$

and

$$Cov(\epsilon_i, \epsilon_j) = 0, i \neq j.$$

Note:

$$E(Y) = X\beta.$$

Characteristics of linear models:

1. Constant variance
2. Independence (uncorrelated) of random errors.
3. Means of the responses independent of the associated variance.

Several questions are raised:

1. What if the variances of the responses are not constant and are dependent on the associated means?
2. What if the means of the responses are always positive (i.e., $E(Y) = X\beta$ might not be sensible).
3. What if the data are categorical or counted?

Nelder and Wedderburn (1972) introduced the generalized linear models. They showed the linearity could be exploited to unify different statistical techniques.