

1.6 Residuals:

There are 3 kinds of residuals which can be used in generalized linear model. They are

- Pearson residual
- Anscombe residual
- Deviance residual

Let

$$\tilde{\mu}_i = g^{-1}(x_i \hat{\beta})$$

and

$$\tilde{\theta}_i = (\mathbf{b}')^{-1}(\tilde{\mu}_i)$$

where $\hat{\beta}$ is the iterated reweighted least squares estimate.

1. Pearson residual:

$$r_{p,i} = \frac{y_i - \tilde{\mu}_i}{\sqrt{\widehat{Var}(Y_i)}} = \frac{y_i - \mathbf{b}'(\tilde{\theta}_i)}{\sqrt{a(\phi)\mathbf{b}''(\tilde{\theta}_i)}}.$$

2. Anscombe residual:

Assume

$$Var(Y) = V(\mu).$$

Define

$$A(x) = \int \frac{1}{V^{1/3}(x)} dx.$$

Then,

$$A(Y_i) - A(\mu_i) \approx A'(\mu_i)(Y_i - \mu_i)$$

and further

$$Var[A(Y_i)] \approx Var[A'(\mu_i)(Y_i - \mu_i)] = [A'(\mu_i)]^2 Var(Y_i).$$

Therefore

$$\sqrt{Var[A(Y_i)]} \approx A'(\mu_i) \sqrt{Var(Y_i)}.$$

The Anscombe residual is

$$r_{a,i} = \frac{A(y_i) - A(\tilde{\mu}_i)}{\sqrt{\widehat{Var}[A(Y_i)]}} = \frac{A(y_i) - A(\tilde{\mu}_i)}{A'(\tilde{\mu}_i) \sqrt{\widehat{Var}(Y_i)}} = \frac{A(y_i) - A[\mathbf{b}'(\tilde{\theta}_i)]}{A'[\mathbf{b}'(\tilde{\theta}_i)] \sqrt{a(\phi)\mathbf{b}''(\tilde{\theta}_i)}}.$$

Example 2 (Poisson distribution, continue):

$Y \sim P(\mu)$. Then,

$$Var(Y) = V(\mu) = \mu$$

and

$$A(x) = \int \frac{1}{V^{1/3}(x)} dx = \int \frac{1}{x^{1/3}} dx = \frac{3x^{2/3}}{2} + c,$$

where c is some constant.

The Anscombe residual is

$$r_{a,i} = \frac{3}{2} \cdot \frac{\left(y_i^{2/3} - \tilde{\mu}_i^{2/3} \right)}{\tilde{\mu}_i^{1/6}} = \frac{3}{2} \cdot \frac{\left(y_i^{2/3} - [b'(\tilde{\theta}_i)]^{2/3} \right)}{[b'(\tilde{\theta}_i)]^{1/6}}.$$

3. Deviance residual:

$$\begin{aligned} r_{a,i} &= sign(y_i - \tilde{\mu}_i) \sqrt{2[y_i(\hat{\theta}_i - \tilde{\theta}_i) + b(\tilde{\theta}_i) - b(\hat{\theta}_i)]} \\ &= sign(y_i - b'(\tilde{\theta}_i)) \sqrt{d_i}, \end{aligned}$$

where

$$\sum_{i=1}^n d_i = D(y_1, y_2, \dots, y_n | \tilde{\mu}_1, \tilde{\mu}_2, \dots, \tilde{\mu}_n).$$