7.3 Dependent observations:

Suppose now that $Cov(Y) = \sigma^2 V(\mu)$, where $V(\mu)$ is a symmetric positive-definite $n \times n$ matrix of known functions $V_{ij}(\mu)$, no longer diagonal. The quasi-score function is

$$U(\beta) = \frac{1}{\sigma^2} D^t V^{-1} (y - \mu),$$

where $D = [D_{ir}]_{n \times p}$ and

$$D_{ir} = \frac{\partial \mu_i}{\partial \beta_r}.$$

The score function has the following properties,

$$E[U(\beta)] = 0, I(\beta) = Cov[U(\beta)] = -E\left[\frac{\partial U(\beta)}{\partial \beta}\right] = \frac{1}{\sigma^2} D^t V^{-1} D.$$

Under suitable limiting conditions, the root $\hat{\beta}$ of the estimating equation

$$U(\hat{\beta}) = \frac{1}{\sigma^2} \hat{D}^t \hat{V}^{-1} (y - \hat{\mu}) = 0$$

is approximately unbiased for β and asymptotically normally distributed with limiting variance

$$Cov(\hat{\beta}) \cong I^{-1}(\beta) = \sigma^2 (D^t V^{-1} D)^{-1}$$

Note:

Block-diagonal covariance matrices arise most commonly in longitudinal studies, in which repeat measurements made on the same subject are usually positively correlated. A well-known application is the generalized estimating equation proposed by Liang and Zeger (1986).