

### **Homework 3:**

1. Please write a program to find all solutions to 7 decimal places of accuracy using Newton-Raphson method for the equation:

$$f(x) = x^3 - 4.5x^2 + 6.5x - 3 = 0.$$

(Hint: plot the function  $f(x)$  to determine the sensible initial values)

2. Write a program to find all solutions to 5 decimal places of accuracy using Newton-Raphson method for the equations:

$$3x^2 - 2y^2 - 1 = 0$$

$$x^2 - 2x + y^2 + 2y - 8 = 0$$

with the starting point  $\begin{bmatrix} -1 \\ 1 \end{bmatrix}$ . Please save the solutions as outputs in a list.

3. Let a random sample  $X_1, X_2, \dots, X_{100}$  from the population with a one-parameter Weibull distribution, i.e., the density being

$$f(x|r) = rx^{r-1}e^{-x^r}, x > 0, r > 0.$$

Please generate 100 observations from Weibull distribution with  $r = 1$  first, then use Newton-Raphson method to find MLE to 5 decimal places of accuracy.

4. Let a random sample  $X_1, X_2, \dots, X_{100} \sim N(\mu, \sigma^2)$ . Please generate 100 observations from  $N(1, 1)$  first, then use Newton-Raphson method to find MLE to 5 decimal places of accuracy.

5. According to central limit theorem,

$$\bar{X} \approx N\left(\mu, \frac{\sigma^2}{n}\right).$$

Please generate the data with sample sizes  $n = 10, 50, 10000$  from Poisson(1). Then, run a simulation study to justify the central limit theorem.

6. Let a random sample  $X_1, X_2, \dots, X_n \sim N(\mu, \sigma^2)$ . Then, one estimator of  $\mu/\sigma^2$  is

$\bar{X}/S^2$ , where  $\bar{X}$  and  $S^2$  are sample mean and sample variance, respectively.

Please write a program to justify the convergence of  $\bar{X}/S^2$  to  $\mu/\sigma^2$  in probability.

7. Let a random sample of  $X_1, \dots, X_n \sim \text{Poisson}(\lambda)$ . Then, two estimators of  $(1 + \lambda)e^{-\lambda}$  are

$$\delta_1 = (1 + \bar{X})e^{-\bar{X}}; \quad \delta_2 = \left(\frac{n-1}{n}\right)^{n\bar{X}} + \bar{X} \left(\frac{n-1}{n}\right)^{n\bar{X}-1}.$$

Please sample 100 data from  $\text{Poisson}(1)$ . The sampling process is repeated 1000 times. Please write a program to do the following:

- (a) Find the averages for the above two estimates.
- (b) Find the averages of the absolute differences between the above two estimates and the true value of the parameter.