Homework 4:

1. Consider the following results for two independent random samples taken from two populations with equal variance.

	Sample 1	Sample 2
Sample Size	$n_1 = 13$	$n_2 = 15$
Sample Mean	$\bar{x}_1 = 13.6$	$\bar{x}_2 = 11.6$
Sample Standard Deviation	$s_1 = 2.2$	$s_2 = 3$

(a) Find the **t-statistic** and **p-value** for the following hypothesis test
$$H_0: \mu_1 \geq \mu_2 + 1 \quad vs. H_1: \mu_1 < \mu_2 + 1.$$

(b) Test the following hypothesis test
$$H_0: \mu_1 = \mu_2 + 3 \quad vs. H_1: \mu_1 \neq \mu_2 + 3$$
by the **confidence interval** method with $\alpha = 0.05$.

(c) Find the **t-statistic** and **p-value** for the following hypothesis test
$$H_0: \mu_1 \leq 13 \quad vs. H_1: \mu_1 > 13.$$

(d) Test the following hypothesis test
$$H_0: \mu_2 = 10 \quad vs. H_1: \mu_2 \neq 10$$
by the **confidence interval** method with $\alpha = 0.01$.

2. 5 observations were selected from each of 3 populations with equal variance.

	Sample 1	Sample 2	Sample 3
Sample mean	30	45	36
Sample variance	6	4	6.5

(a) Compute the between-treatments and within-treatments estimates of the population variance.

(b) Find the F statistic and test $H_0: \mu_1 = \mu_2 = \mu_3$ at $\alpha = 0.01$.

3. Suppose we have the following data:

1.86	0.51	-1.14	0.33	0.17	0.67	-1.69	1.02	3.32	-0.20
-0.27	1.00	0.39	-1.80	0.68	0.40	-2.50	-0.62	-0.49	1.85

Please write a program to do the following:

(a) qq t plot with 10 degrees of freedom.

(b) qq normal plot with $\mu = 0, \sigma^2 = 1.3$.

Put the above 2 plots in the same Figure.

4. Please generate 1000 data from a normal distribution with $\mu = 2$, $\sigma^2 = 4$. Please write a program to do the following:

(a) qq-t plot for the generated data with 15 degrees of freedom.

(b) qq-t plot for the generated data with 2 degrees of freedom.

(c) qq normal plot for the generated data with $\mu = 3, \sigma^2 = 1$.

Put the above 3 plots in the same Figure.

5. (a) The following data have been collected for a sample from a normal population

80	110	100	140	120

Find the 99% confidence interval for population mean μ and the t-statistic for $H_0: \mu = 110$ $vs. H_1: \mu \neq 110$.

(b) Consider the following data for two random samples (independent samples) taken from two normal populations with equal variances.

Sample 1	8	10	5	5	5	9
Sample 2	6	6	4	5	2	7

Find the *t-statistic and its p-value* for.

$$H_0: \mu_1 \geq \mu_2 + 3 \ \ vs. H_1: \mu_1 < \mu_2 + 3,$$

where μ_1 and μ_2 are the means of population 1 and population 2, respectively.

6. The data were selected from each of three normal populations with equal variances. The data obtained follow.

Observation	Sample 1	Sample 2	Sample 3
1	32	44	33
2	30	43	36
3	30	44	35
4	26	46	40
5	32	48	
Sample mean	30	45	36
Sample variance	6	4	8.67

At $\alpha = 0.01$, find the F statistic and test the null hypothesis that 3 population means are equal?

7. The following are the number of wrong answers for the number of the students.

Number of wrong answers	0	1	2	3	4
Number of the students	3	8	9	9	3

Suppose X is the random variable representing the number of wrong answers. Please test X is distributed as $Binomial(4, 0.45)$ with $\alpha = 0.05$.

8. Please generate 1000 data from a t distribution with the degree of freedom equal to 10 first. Then,

(a) test if the data is distributed as the standard normal distribution with $\alpha = 0.05$.

(b) test if the data is distributed as the t distribution with the degree of freedom equal to 10 at $\alpha = 0.05$.