Homework 5:

1. For the following data

Sample	59	56	47	46	55	54	48

Please perform Wilcoxon sign rank test for testing

$$H_0: M \geq 51 \ \ vs. H_1: M < 51$$

at $\alpha = 0.05$, where M is the median of the population. Please also compute the Wilcoxon sign rank statistic not using the splus command wilcox.test.

2. For the following data

Sample 1	59	56	47	46	55	54	48
Sample 2	63	49	60	54	56	55	

Please perform Wilcoxon rank sum test for testing

$$H_0: M_1 = M_2 \ \ vs. H_1: M_1 \neq M_2,$$

at $\alpha = 0.05$, where M_1 and M_2 are the medians of the two populations. Please also compute the Wilcoxon rank sum statistic not using the splus command wilcox.test.

3. Here is a set of data with the model

$$Y_i = \beta_0 + \beta_1 X_{i1} + \beta_2 X_{i2} + \epsilon_i, i = 1, \cdots, 7, \epsilon_i \sim N(0, \sigma^2),$$

Y_i	5	10	10	25	10	30	20
X_{i1}	-1	1	1	0	-1	-1	1
X_{i2}	5	0	-3	-4	-3	0	5

(a) Find the estimated variance-covariance matrix of the least squares estimate.

(b) Find the t statistic for testing $H_0: \beta_2 \leq 1$ and the associated p-value.

(c) Find F statistic to test $H_0: \beta_1 = \beta_2 = 0$ and the associated p-value.

(d) Find F statistic to test $H_0: \beta_1 = 0$ and the associated p-value.

(e) Find F statistic to test $H_0: \beta_1 = \beta_2$ and the associated p-value.

(f) Find the 95% confidence interval for $E(\widehat{Y}_8)$ at $X_8 = [1 \quad 0 \quad 5]$.

Note: Please not use the function "lm" and use matrix manipulations only.

4. Here is a set of data with the model

$$Y_i = \beta_0 + \beta_1 X_{i1} + \beta_2 X_{i2} + \epsilon_i, i = 1, \cdots, 4, \epsilon_i \sim N(0, \sigma^2)$$

Y_i	1	3	5	7
X_{i1}	-1	1	1	-1
X_{i2}	1	0	0	-1

(a) Find the least squares estimate and R^2.

(b) Find the t statistic and p-value to test $H_0: \beta_1 \le 0$.

(c) Find F statistic to test $H_0: \beta_1 = \beta_2 = 0$ at $\alpha = 0.05$.

(d) Find F statistic and p-value to test $H_0: 3\beta_0 + 2\beta_1 + 4\beta_2 = 0$.

(e) Find the 95% confidence interval for $E(\hat{Y}_5)$ at $X_5 = [1 \quad -1 \quad 0]$.

5. Please generate the regular data from the model

$$Y_i = 1 + 2X_{i1} + 3X_{i2} + \epsilon_i, i = 1, \cdots, 100, \epsilon_i \sim N(0, 4),$$

where both X_{i1} and X_{i2} are generated from a standard normal random variable. Then, the least square estimate can be obtained. By repeating the above process 1000 times, please find mean absolute difference of the sum of the least squares estimates and the sum of the true values of the parameters.