

II. Computing

4.7. Computations in calculus

1. Differentiation:

Example:

Please find the derivative

$$\frac{de^{\sin(x^2)}}{dx} \text{ at } x = 1.$$

Example (Splus):

D(expression(exp(sin(x^2))), "x")

x=1

eval(D(expression(exp(sin(x^2))), "x"))

by numerical approximation

ep=2e-8

xe=x+ep

(exp(sin(xe^2))-exp(sin(x^2)))/ep

2. Integration:

Two commonly used approximation for the integral $\int_a^b f(x) dx$.

(a) Trapezoidal Rule:

$$S(n) = \Delta x \left[\frac{f(a)}{2} + f(a + \Delta x) + f(a + 2\Delta x) + \dots + f(a + (n - 1)\Delta x) + \frac{f(b)}{2} \right]$$

where $\Delta x = (b - a)/n$.

Special case:

The integral $\int_0^1 f(x) dx$ can be approximated by

$$S(n) = \frac{1}{n} \left[\frac{f(0)}{2} + f\left(\frac{1}{n}\right) + f\left(\frac{2}{n}\right) + \dots + f\left(\frac{n-1}{n}\right) + \frac{f(b)}{2} \right]$$

(b) Simpson's Rule:

$$S(n) = \frac{\Delta x}{3} [f(a) + 4f(a + \Delta x) + 2f(a + 2\Delta x) + 4f(a + 3\Delta x) + 2f(a + 4\Delta x) + \dots + 4f(a + (n-1)\Delta x) + f(b)]$$

where $\Delta x = (b - a)/n$ and n is even.

Special case:

The integral $\int_0^1 f(x) dx$ can be approximated by

$$S(n) = \frac{1}{3n} \left[f(0) + 4f\left(\frac{1}{n}\right) + 2f\left(\frac{2}{n}\right) + \dots + 4f\left(\frac{n-1}{n}\right) + f(1) \right].$$

Example:

Please approximate the integral $\int_0^2 e^{x^2} dx = 16.4526278$ by the above approximations by using $S(100)$.

Example (Splus):

Trapezoidal Rule:

```
trapAppro=function(n,a,b)
{
  dx=(b-a)/n
  seqn=seq(a,b,by=dx)
  sn=rep(0,length(seqn))
  sn[2:n]=dx*exp(seqn[2:n]^2)
  sn[c(1,n+1)]=(dx/2)*exp(seqn[c(1,n+1)]^2)
  sum(sn)
}
trap100=trapAppro(100,0,2)
```

Simpson's Rule:

```
simpAppro=function(n,a,b)
{
  dx=(b-a)/n
  seqn=seq(a,b,by=dx)
  sn=rep(0,n+1)
  s1=seq(2,n,by=2)
```

```
sn[s1]=(4*dx/3)*exp(seqn[s1]^2)
s2=seq(3,n-1,by=2)
sn[s2]=(2*dx/3)*exp(seqn[s2]^2)
sn[c(1,n+1)]=(dx/3)*exp(seqn[c(1,n+1)]^2)
sum(sn)
}
simp100=simpAppro(100,0,2)
tValue=16.4526278
c(trap100-tValue,simp100-tValue)
options(digits=9)
trapAppro(500,0,2)
simpAppro(500,0,2)
```