Review 2

92.4.8

- **Interval estimate, hypothesis tests, and p-value of the population mean difference $\mu_1 - \mu_2$.**
- **Two methods, independent samples and matched samples, can be used for the population mean difference $\mu_1 - \mu_2$.**

Example:

To determine the effectiveness of a new weight control diet, ten randomly selected students observed the diet for 4 weeks with the results shown below.

Dieter	Weight (before)	Weight (after)
A	*138*	*135*
B	*151*	*147*
C	*129*	*132*
D	*125*	*127*
E	*168*	*155*
F	*139*	*131*
G	*152*	*144*
H	*140*	*142*
I	*137*	*137*
J	*180*	*180*

We like to test the hypothesis $H_0 : \mu_1 = \mu_2$, where μ_1 and μ_2 are the mean weights of the students before and after taking the weight control diet, respectively.

(a) For $\alpha = 0.1$, test the above hypothesis using the classical hypothesis test.

(b) For $\alpha = 0.05$, please use the confidence interval method to test the above hypothesis.

(c) For $\alpha = 0.2$, please use p-value to test the above hypothesis.

[solution:]

(a)

d_1	d_2	d_3	d_4	d_5	d_6	d_7	d_8	d_9	d_{10}
3	***4***	***-3***	***-2***	***13***	***8***	***8***	***-2***	***0***	***0***

Therefore, $\bar{d} = 2.9, s_d = 5.322$. Thus,

$$t = \frac{\bar{d}}{\left(s_d / \sqrt{n}\right)} = \frac{2.9}{\left(5.322 / \sqrt{10}\right)} = 1.723 \Rightarrow |t| = 1.723 < 1.833 = t_{9,0.05} = t_{n-1,\alpha/2}.$$

We do ***not*** reject H_0.

(b)

A 95% confidence interval for $\mu_1 - \mu_2$ is

$$\bar{d} \pm t_{n-1,\alpha/2} \frac{s_d}{\sqrt{n}} = 2.9 \pm t_{9,0.025} \cdot \frac{5.322}{\sqrt{10}} = 2.9 \pm 2.262 \cdot \frac{5.322}{\sqrt{10}} = 2.9 \pm 3.807 = [-0.907, 6.707].$$

Since $0 \in [-0.907, 6.707]$, we do ***not*** reject H_0.

(c)

$$p-value = P(|T(n-1)| > t) = P(|T(9)| > 1.723) < 0.2 = P(|T(9)| > 1.383).$$

Therefore, we reject H_0.

2. Consider the following results for two samples randomly taken from two populations.

	Sample 1	**Sample 2**
Sample size	**64**	**49**
Mean	**1150**	**921**
Standard deviation	**90**	**65**

Let μ_1 and μ_2 be the population means.

(a) For $\alpha = 0.05$, test $H_0 : \mu_1 - \mu_2 \le 200$ using the classical hypothesis test.

(b) For $\alpha = 0.01$, please use p-value to test $H_0 : \mu_1 - \mu_2 \le 200$.

(c) For $\alpha = 0.05$, please use the confidence interval method to test the hypothesis $H_0 : \mu_1 - \mu_2 = 200$.

[solution:]

(a)

$\bar{x}_1 = 1150, \bar{x}_2 = 921, s_1 = 90, s_2 = 65, n_1 = 64, n_2 = 49, c = 200, \alpha = 0.05$.

Then,

$$z = \frac{\bar{x}_1 - \bar{x}_2 - c}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} = \frac{1150 - 921 - 200}{\sqrt{\frac{90^2}{64} + \frac{65^2}{49}}} = 1.99 > z_\alpha = z_{0.05} = 1.645$$

Therefore, we reject H_0.

(b)

$$p-value = P(Z > z) = P(Z > 1.99) = 0.0233 > \alpha = 0.01.$$

Therefore, we do not reject H_0.

(c)

A 95% confidence interval for $\mu_1 - \mu_2$ is

$$\begin{aligned}(\bar{x}_1 - \bar{x}_2) \pm z_{\alpha/2}\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} &= (1150 - 921) \pm z_{0.025}\sqrt{\frac{90^2}{64} + \frac{65^2}{49}} \\ &= 229 \pm (1.96 \cdot 14.587) = [200.41, 257.59]\end{aligned}.$$

Since $200 \notin [200.41, 257.59]$, we reject H_0.