

### 3.6. The weighted mean and grouped data:

#### 1. Weighted mean:

$$\bar{x}_w = \frac{\sum_{i=1}^n w_i x_i}{\sum_{i=1}^n w_i}.$$

#### Note:

When data values vary in importance, the analyst must choose the weight that best reflects the importance of each data value in the determination of the mean.

#### Example 6:

The following are 5 purchases of a raw material over the past 3 months.

Purchase	Cost per Pound (\$)	Number of Pounds
1	3.00	1200
2	3.40	500
3	2.80	2750
4	2.90	1000
5	3.25	800

Find the mean cost per pound.

[Solution:]

$$w_1 = 1200, w_2 = 500, w_3 = 2750, w_4 = 1000, w_5 = 800$$

and

$$x_1 = 3.00, x_2 = 3.40, x_3 = 2.80, x_4 = 2.90, x_5 = 3.25.$$

Then,

$$\begin{aligned}\bar{x}_w &= \frac{\sum_{i=1}^n w_i x_i}{\sum_{i=1}^n w_i} = \frac{1200 \cdot 3 + 500 \cdot 3.4 + 2750 \cdot 2.8 + 1000 \cdot 2.9 + 800 \cdot 3.25}{1200 + 500 + 2750 + 1000 + 800} \\ &= 2.96\end{aligned}$$

#### 2. Grouped data:

##### Population Mean for Grouped Data:

$$\mu_g = \frac{\sum_{k=1}^m F_k M_k}{\sum_{k=1}^m F_k} = \frac{\sum_{k=1}^m F_k M_k}{N},$$

where

$M_k$ : the midpoint for class  $k$ ,

$F_k$ : the frequency for class  $k$  in the population,

$N = \sum_{k=1}^m F_k$ : the population size.

### Sample Mean for Grouped Data:

$$\bar{x}_g = \frac{\sum_{k=1}^m f_k M_k}{\sum_{k=1}^m f_k} = \frac{\sum_{k=1}^m f_k M_k}{n},$$

where

$f_k$ : the frequency for class  $k$  in the sample,

$n = \sum_{k=1}^m f_k$ : the sample size.

### Population Variance for Grouped Data:

$$\sigma_g^2 = \frac{\sum_{k=1}^m F_k (M_k - \mu_g)^2}{N}$$

### Sample Variance for Grouped Data:

$$s_g^2 = \frac{\sum_{k=1}^m f_k (M_k - \bar{x}_g)^2}{n - 1} = \frac{\sum_{k=1}^m f_k M_k^2 - n \bar{x}_g^2}{n - 1}$$

#### Example 3 (continue):

Based on the frequency table obtained in (g), compute the mean and variance for the grouped data. Compare with the results in (a) and (b).

[Solution:]

$$\bar{x}_g = \frac{5.5 \cdot 3 + 15.5 \cdot 4 + 25.5 \cdot 4 + 35.5 \cdot 4}{15} = 21.5$$

and

$$\begin{aligned} s_g^2 &= \frac{3 \cdot (5.5 - 21.5)^2 + 4 \cdot (15.5 - 21.5)^2 + 4 \cdot (25.5 - 21.5)^2 + 4 \cdot (35.5 - 21.5)^2}{14} \\ &= 125.71 \end{aligned}$$

$$\text{Thus, } s_g = \sqrt{s_g^2} = 11.212.$$

The group mean and the group standard deviation are close to the original mean and standard deviation.

#### Example 7:

The following are the frequency distribution of the time in days required to complete year-end audits:

Audit Time (days)	Frequency
10 – 14	4
15 – 19	8
20 – 24	5
25 – 29	2
30 – 34	1

What is the mean and the variance of the audit time?

[Solution:]

$$f_1 = 4, f_2 = 8, f_3 = 5, f_4 = 2, f_5 = 1.$$

$$n = f_1 + f_2 + f_3 + f_4 + f_5 = 4 + 8 + 5 + 2 + 1 = 20$$

and

$$M_1 = 12, M_2 = 17, M_3 = 22, M_4 = 27, M_5 = 32.$$

Thus,

$$\bar{x}_g = \frac{\sum_{k=1}^5 f_k M_k}{\sum_{k=1}^5 f_k} = \frac{4 \cdot 12 + 8 \cdot 17 + 5 \cdot 22 + 2 \cdot 27 + 1 \cdot 32}{4 + 8 + 5 + 2 + 1} = 19$$

and

$$\begin{aligned}
 s_g^2 &= \frac{\sum_{k=1}^5 f_k (M_k - \bar{x}_g)^2}{n - 1} \\
 &= \frac{4 \cdot (12 - 19)^2 + 8 \cdot (17 - 19)^2 + 5 \cdot (22 - 19)^2 + 2 \cdot (27 - 19)^2 + 1 \cdot (32 - 19)^2}{20 - 1} \\
 &= 30
 \end{aligned}$$