4.3. Some Basic Relationships of Probability:

A^c: **The complement of A, the event containing all sample points that are not in A.**

$A \cup B$: **The union of A and B, the event containing all sample points belonging to A or B or both.**

$A \cap B$: **The intersection of A and B, the event containing all sample points belonging to both A and B.**

Example 10 (continue):

$$E_1 = \{2,4,6\}, E_2 = \{1,3,5\}, E_3 = \{1,2,3\}.$$

Then

$$E_1^c = E_2,$$

$$E_1 \cup E_3 = \{1,2,3,4,6\} \equiv even\ \ points\ \ or\ \ points \le 3,$$

$$E_1 \cap E_3 = \{2\} \equiv even\ \ points\ \ and\ \ points \le 3,$$

Note:

Two events having no sample points in common is called mutually exclusive events. That is, if A and B are mutually exclusive events, then

$$A \cap B = \phi \equiv empty\ \ set.$$

Example 10 (continue):

$$E_1 = \{2,4,6\}, E_2 = \{1,3,5\}.$$

$$E_1 \cap E_2 = \phi \Longrightarrow E_1\ \ and\ \ E_2\ \ are\ \ mutually\ \ exclusive\ \ events.$$

Results:

1. For any event A, $\mathrm{P}(A) = 1 - P(A^c)$, i.e., $\mathrm{P}(A) + P(A^c) = 1$.

2. If A and B are mutually exclusive events, then

$$P(A \cap B) = 0$$

and

$$P(A \cup B) = P(A) + P(B).$$

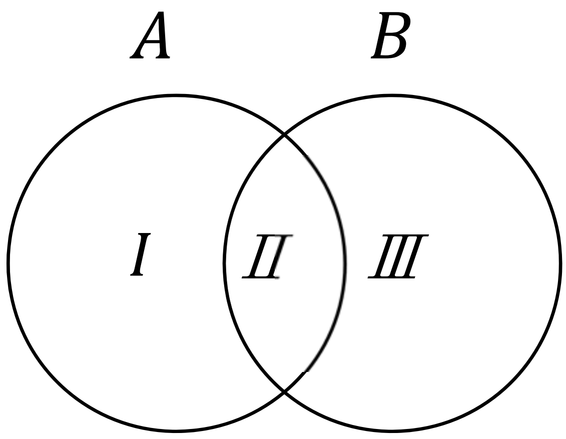
3. For any two events A and B,

$$P(A \cup B) = P(A) + P(B) - P(A \cap B),$$

i.e.,

$$P(A \cup B) + P(A \cap B) = P(A) + P(B).$$

[Intuition of Result 3]:



$$II = A \cap B, A = I \cup II, B = II \cup III.$$

Then

$$\begin{aligned} P(A \cup B) &= P(I) + P(II) + P(III) \\ &= [P(I) + P(II)] + [P(II) + P(III)] - P(II) \\ &= P(I \cup II) + P(II \cup III) - P(II) \\ &= P(A) + P(B) - P(A \cap B) \end{aligned}$$

Example 10 (continue):

Roll a fair dice. Let $S = \{1,2,3,4,5,6\}$, $E_1 = \{2,4,6\}, E_2 = \{1,3,5\}$ **and** $E_3 = \{1,2,3\} \equiv points \leq 3.$

1. $P(E_2) = P(\{1,3,5\}) = P(\{2,4,6\}^c) = P(E_1^c) = 1 - P(E_1) = 1 - \frac{1}{2} = \frac{1}{2}.$

2. $P(E_1 \cap E_2) = 0, P(\Omega) = P(E_1 \cup E_2) = P(E_1) + P(E_2) = \frac{1}{2} + \frac{1}{2} = 1.$

3. $P(E_1 \cup E_3) = P(\{1,2,3,4,6\}) = \frac{5}{6}$. **We can also use the addition law, i.e.,**

$$\begin{aligned} P(E_1 \cup E_3) &= P(E_1) + P(E_3) - P(E_1 \cap E_3) \\ &= P(\{2,4,6\}) + P(\{1,2,3\}) - P(\{2\}) \\ &= \frac{1}{2} + \frac{1}{2} - \frac{1}{6} = \frac{5}{6}. \end{aligned}$$

Example 11:

Assume you are taking two courses this semester (S and C). The probability that you will pass course S is 0.835**, the probability that you will pass both courses is** 0.276**. The probability that you will pass at least one of the courses is** 0.981**. What is the probability that you will pass course C?**

[Solution:]

Let A be the event of passing course S and B be the event of passing course C. Thus,

$$P(A) = 0.835, P(A \cap B) = 0.276, P(A \cup B) = 0.981$$

Method 1:

$$P(B) = P(A \cup B) + P(A \cap B) - P(A) = 0.981 + 0.276 - 0.835 = 0.422.$$

Method 2:

Since

$$P(A^c \cap B) = P(A \cup B) - P(A) = 0.981 - 0.835 = 0.146,$$

$$P(B) = P(A \cap B) + P(A^c \cap B) = 0.276 + 0.146 = 0.422.$$