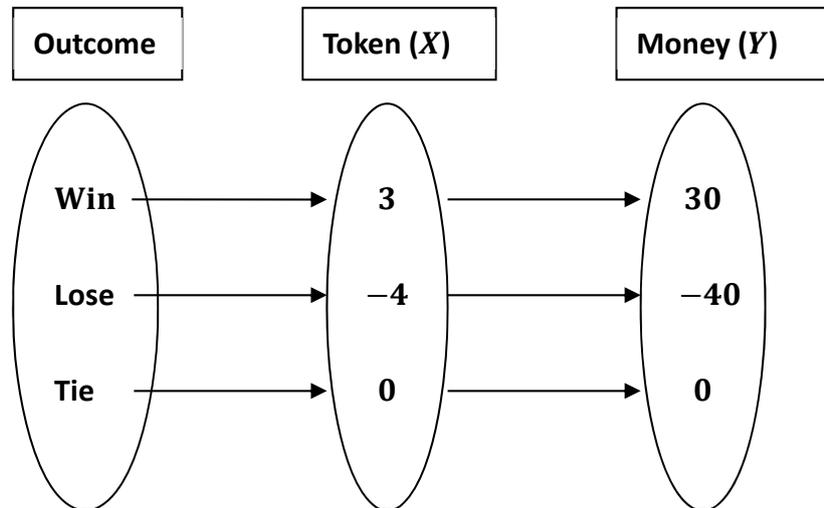


Chapter 5 Discrete Probability Distributions

5.1. Random variable:

Example 1:

Suppose we gamble in a casino and the possible result is as follows.



In this example, the sample space is $S = \{Win, Lose, Tie\}$, containing 3 outcomes. X is the quantity representing the token obtained or lose under different result while Y is the one representing the money obtained or lost. In the above example, X and Y can provide a numerical summary corresponding to the experimental outcome. A formal definition for these numerical quantities is in the following.

Definition (random variable): A random variable is a numerical description of the outcome of an experiment.

Note: A random variable X is defined as a **real-valued** function on **the sample space**, i.e.,

$$X: S \rightarrow R.$$

Example 1 (continue):

In the previous example,

X : the random variable representing the token obtained or lose corresponding to different outcomes.

Y : the random variable representing the money obtained or lose corresponding to different outcomes.

X has 3 possible values corresponding to 3 outcomes

$$\Rightarrow X(Win) = 3, X(Lose) = -4, X(Tie) = 0.$$

Y has 3 possible values corresponding to 3 outcomes

$$\Rightarrow Y(\text{Win}) = 30, Y(\text{Lose}) = -40, Y(\text{Tie}) = 0.$$

Note that $Y = 10X$ since

$$Y(\text{Win}) = 30 = 10X(\text{Win}), Y(\text{Lose}) = -40 = 10X(\text{Lose}), \\ Y(\text{Tie}) = 0 = 10X(\text{Tie})$$

That is, Y is 10 times of X under all possible experimental outcomes.

There are **two types of random variables**. They are:

Discrete random variable: A quantity assumes either a finite number of values or an infinite sequence of values, such as 0, 1, 2, ...

Continuous random variable: A quantity assumes any numerical value in an interval or collection of intervals, such as time, weight, distance, and temperature.

Example 2:

Let the sample space

$$S = \{x \text{ hour} \mid x \text{ is the delay time for a flight}, 0 \leq x \leq 1\}.$$

Let X be the random variable representing the delay flight time, defined as

$$X(x \text{ hour}) = x, 0 \leq x \leq 1.$$

For example, $X = 0.5$ corresponds to the outcome that the flight time is 0.5 hour (30 minutes) late.