5.1. Experiments, Counting Rules, and Probabilities

Experiment: **any process that generates well-defined outcomes.**

Example:

Experiment	*Outcomes*
Toss a coin	**Head, Tail**
Roll a dice	**1, 2, 3, 4, 5, 6**
Play a football game	**Win, Lose, Tie**
Rain tomorrow	**Rain, No rain**

Sample Space: **the set of all experimental outcomes, denoted by *S***

Example:

Experiment	*Sample Space*
Toss a coin	***S*={Head, Tail}**
Roll a dice	***S*={1, 2, 3, 4, 5, 6}**
Play a football game	***S*={Win, Lose, Tie}**
Rain tomorrow	***S*={Rain, No rain}**

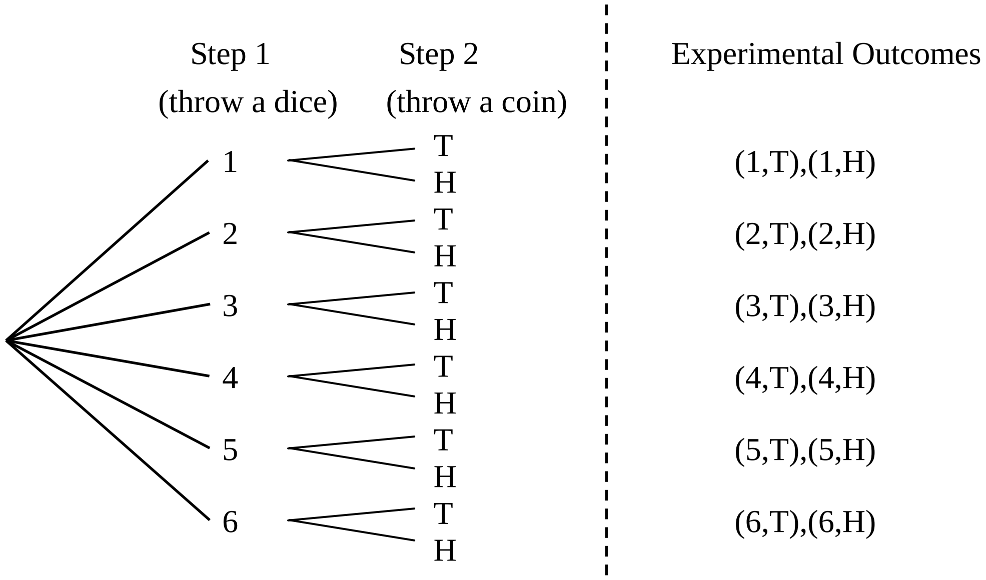
Counting Rules: **the rules for counting the number of the experimental outcomes.**

We have the following counting rules:

- Multiple Step Experiment:
- Permutations
- Combinations

1. Multiple Step Experiment:

Example:



$\Rightarrow$ $S = \{(1,T),(1,H),(2,T),(2,H),(3,T),(3,H),(4,T),(4,H),(5,T),(5,H),(6,T),(6,H)\}$

$\Rightarrow$ The total number of experimental outcomes=12 $= 6 \cdot 2$

Counting rule for multiple step experiments:

If there are k-steps in an experiment which there are n_1 possible outcomes on the first step, n_2 possible outcomes on the second step, and so on, then the total number of experimental outcomes is given by $n_1 \cdot n_2 \cdots n_k$.

2. Permutations:

***n* objects are to be selected from a set of *N* objects, where the order is important.**

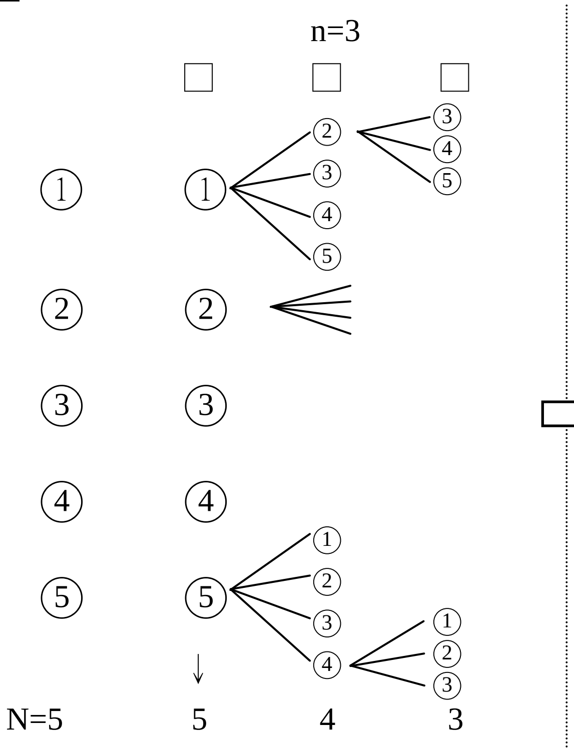
Example:

Suppose we take 3 balls from 5 balls, 1, 2, 3, 4 and 5. Then, ① ② ③

⇒ two permutations (different orders)

② ① ③

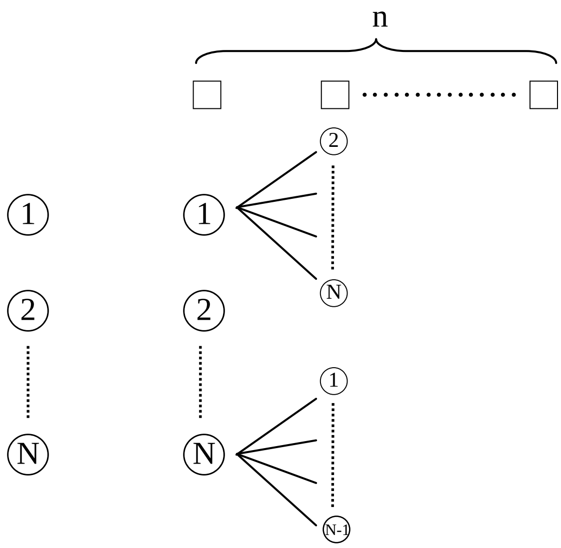
Example:



$$5 \cdot 4 \cdot 3$$

$$\Rightarrow \underbrace{N \cdot (N-1) \cdot \cdots\cdots \cdot (N-n+1)}_{n}$$

Example:



$$\underbrace{N \cdot (N-1) \cdot (N-2) \cdot \cdots\cdots \cdot [N-(n-1)]}_{n} = \frac{N!}{(N-n)!}$$

Counting rule for permutation:

As n objects are taken from N objects, then the total number of permutations is given by

$$P_n^N = \frac{N!}{(N-n)!} = (N-n+1)(N-n+2)\cdots N$$

where $N! = 1\cdot 2\cdot 3\cdots N$ and $0! = 1.$

3. Combinations:

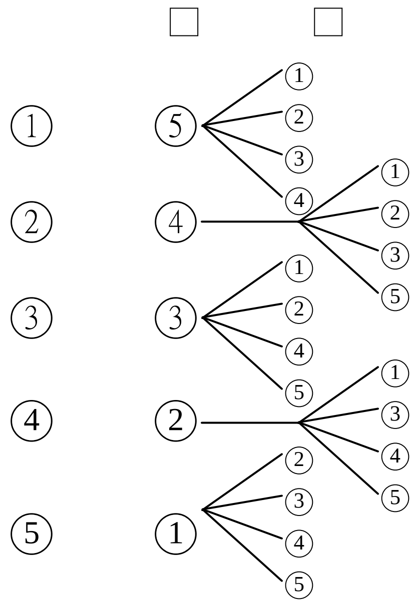
***n* objects are to be selected from a set of *N* objects, where the order is not important.**

Example:

□	□	□
①	②	③
①	③	②
②	①	③
②	③	①
③	①	②
③	②	①

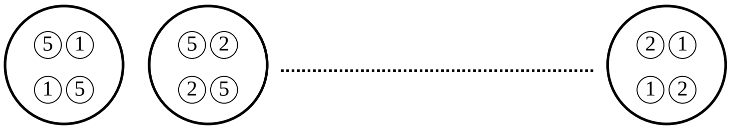
} 1 combination, but 6 permutations.

Example:



$$\Rightarrow P_2^5 = 5 \cdot 4 = 20$$

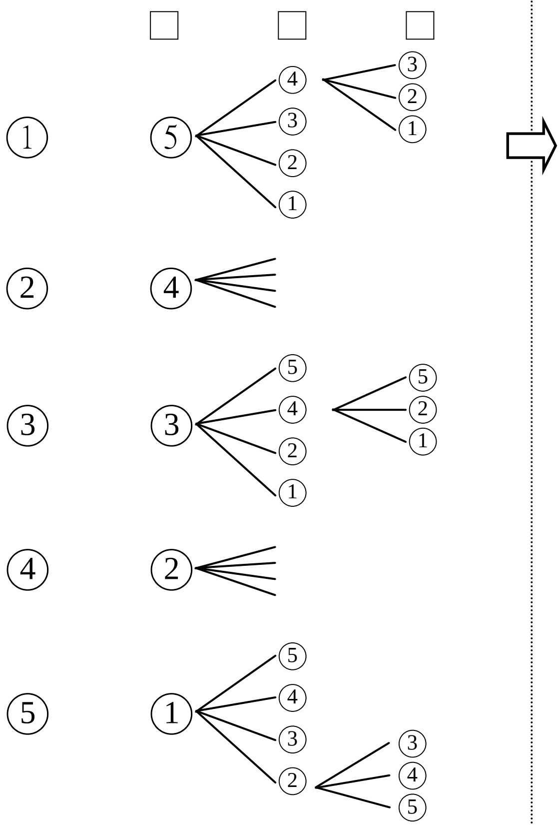
$$C_2^5 = \frac{P_2^5}{2!} = 10$$



⇒ **10 combinations**

20 permutations

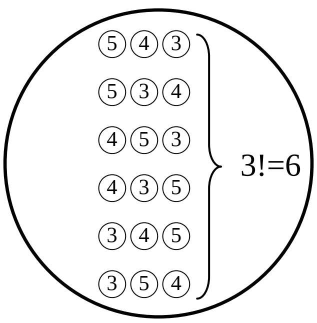
Example:



$$P_3^5 = 5 \cdot 4 \cdot 3 = 60$$

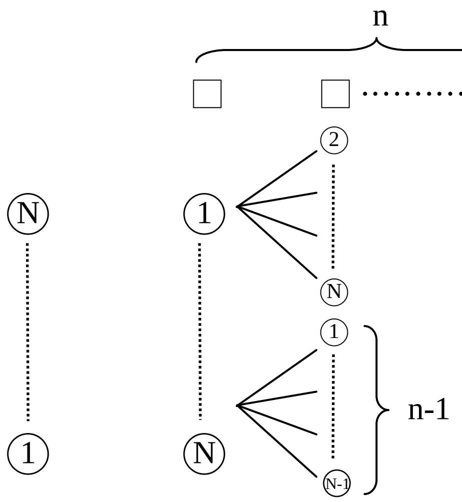
$$C_3^5 = \frac{P_3^5}{3!} = \frac{P_3^5}{6} = 10$$

5 • 4 • 3



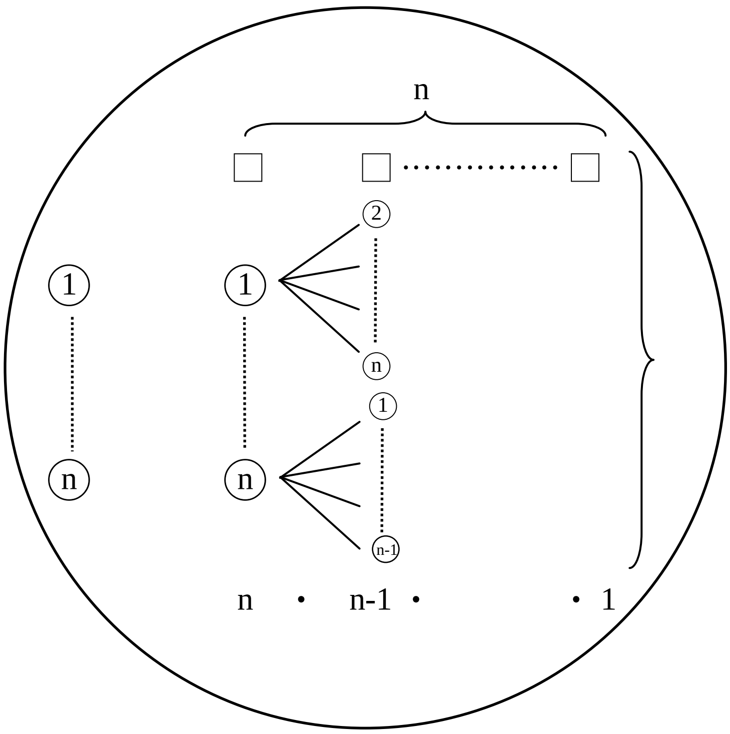
⇒ **1 combination, total 10 combinations.**

Example:



$$\Rightarrow P_n^N \quad \text{permutations}$$

$$C_n^N = \frac{P_n^N}{n!}$$



$$\Rightarrow P_n^n = \frac{n!}{(n-n)!} = \frac{n!}{0!} = n!$$

$\Rightarrow$ 1 combination

Counting rule for combination:

As n objects are taken from N objects, then the total number of combinations is given by

$$C_n^N = \binom{N}{n} = \frac{N!}{n!(N-n)!} = \frac{P_n^N}{n!}$$

Online Exercise:

Exercise 5.1.1
Exercise 5.1.2