Final

2021. 1. 12

1. (40%) Let X be a discrete random variable with the following probability distribution:

$$f_X(i) = \frac{100i^2}{3c^3}, i = 1, 2, 3, 4$$

and $f_X(i) = 0$, otherwise, where c is some constant.

(a) Compute c.

(b) Compute $P(1 \le X < 3)$ and $P(-2 \le X < 2)$.

(c) Let A denote the event that the values of X is greater than 1.2, i.e., $X > 1.2$, and let B denote the event that the values of X is smaller than 3.4, i.e., $X < 3.4$. Find the conditional probability $P(B|A)$, i.e., $P(X < 3.4|X > 1.2)$.

(d) Compute $E(X)$.

(e) Compute $Var(X)$.

2. (40%) Given that Z is a standard normal random variable and X is a normal random variable with mean 1 and standard deviation 3.

(a) $P(-2.5 \le Z < 1.5)$

(b) $P(Z > 2)$.

(c) $P(Z > c) = 0.9495$, find c.

(d) $P(X < 4)$.

(e) $P(-5 \le X \le 2c + 1) = 0.9544$, find c.

3. (20%) On the average, the time between arrivals of vehicles at a particular intersection is 0.5 *minute*. Please use Poisson distribution and exponential density to answer the following questions.

(a) What is the probability of 45 *or more seconds* between vehicle arrivals?

(b) What is the probability that the arrival time between vehicles is between 2 *minutes* and 3 *minutes*?

(c) What is the probability that there are 0 *arrivals* within 2 *minutes*?

(d) What is the probability that there are 3 *arrivals* within 3 *minutes*?

4. (20%)

(a) A population has a mean 100 and a standard deviation of 30. Suppose a sample of size $n = 3600$ is taken. What is the probability that a sample mean will be within ± 1 of the population mean, i.e. $P(|\bar{X} - \mu| \le 1)$?

(b) Suppose we have a population of 36 elements

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11	12	13	14	15	16	17	18	19
21	23	5	7	29	2	24	6	38
3	6	27	28	29	0	2	31	8
42	34	6	8	9	1	3	5	47

Suppose the first row of the table of random number is

58683 93108 80714 15141 63271 13154 71744 59986 79945 51102

Please use ***simple random sampling*** methods to obtain a sample of 3 elements.

5. (20%)

(a) The cholesterol level for an adult male of a specific racial group is normally distributed with a mean of 5.2 mmol/L and a standard deviation of 0.65 mmol/L. Which cholesterol level is less by 2.5% of the population?

(b) In your pocket you have 3 dimes (coins of 10 cents) and 2 quarters (coins of 25 cents). You grab at random 2 coins from your pocket. Let X be the random variable representing the amount of money you will grab. Find the probability distribution function of X.

Equations:

$$Poisson:\ f_X(x) = \frac{e^{-\mu}\mu^x}{x!}, x = 0, 1, 2, \cdots$$

$$X \sim N(\mu, \sigma^2) \Longrightarrow \frac{X-\mu}{\sigma} = Z \sim N(0,1)$$

$$\bar{X} \approx N\left(\mu, \sigma_{\bar{X}}^2\right) \Longrightarrow \frac{\bar{X}-\mu}{\sigma_{\bar{X}}} = \frac{\bar{X}-\mu}{\left(\sigma/\sqrt{n}\right)} \approx Z \sim N(0,1)$$