Quiz 2

2020. 12. 22

1. (15%) An oil company has purchased an option on land in Alaska. Preliminary geologic studies have assigned the following prior probabilities.

$$P(high-quality\ \ oil)=0.1, P(medium-quality\ \ oil)=0.1,$$
$$P(no\ \ oil)=0.8.$$

After 200 feet of drilling on the first well, a soil test is taken. The probabilities of finding particular type of soil identified by the test follow.

$$P(soil|high-quality\ \ oil)=0.9, P(soil|medium-quality\ \ oil)=0.8,$$
$$P(soil|no\ \ oil)=0.05.$$

Compute the posterior (conditional) probabilities of finding oil, given finding particular type of *soil* ?.

2. (40%) Let X be a discrete random variable with the following probability distribution:

$$f_X(i)=\frac{i^2}{2c^2}, i=1,2,3,6$$

and $f_X(i)=0$, otherwise, where c is some constant.

(a) Compute *c*.

(b) $P(X>2)$.

(c) the conditional probability $P(X>2.1|X<4.5)$, i.e., $P(A|B)$, where event *A* is $X>2.1$ and event *B* is $X<4.5$.

(d) Compute $E(X)$.

(e) Compute $Var(X)$.

3. (30%)

(a) (10%) Consider a binomial random variable X with $n=5$ and $p=0.2$.

(i) Find $P(X\geq 1)$.

(Ii) Find $P(X=4.5\ \ or\ \ X=3)$.

(b) (10%) A retailer of electronic equipment received 20 VCRs from the manufacturer. 2 VCRs were damaged in the shipment. The retailer sold 8 VCRs to some customer. Let the random variable *X* be the number of damaged VCRS that the customer received. What is the probability that the customer received 7 good VCRs, i.e., $P(X=1)$.

(c) (10%) Two sport teams play a series consisting of at most 5 games until one of the two teams A or B has won 3 games. Suppose the probability that team A will win any game is 0.5, i.e., the probability for team B to win being 0.5. Assuming that the results of the various games are independent of each other. Let the random variable *X* be the number of games in this series. What is the probability distribution function of *X*?

4. (30%) Given that Z is a standard normal random variable and $X \sim N(1, 9)$ is a normal random variable.
   (a) $P(-1.5 \leq Z < 2.5)$
   (b) $P(Z > 0.65)$.
   (c) $P(Z < c) = 0.4495$, find c.
   (d) $P(-2 \leq X < 7)$.
   (e) $P(1 \leq X \leq c) = 0.4901$, find c.

5. (20%) On the average, there are ***10 customers*** at a particular restaurant within ***1 hour***. Please use Poisson distribution and exponential density to answer the following questions.
   (a) Let the random variable *X* be the number of customers within ***8 hours***. Find ***the variance*** of the random variable *X.*
   (b) What is the probability that there is ***no customer*** within ***30 minutes***?
   (c) What is the probability that there are ***at least 1 customer*** within ***2 hour***?
   (d) What is the probability that the arrival time between customers is between ***15 minutes*** and ***30 minutes***?