Chapter 7:
- Compute the standard error and the probability of the sample mean $\bar{x}$ or the sample proportion $\bar{p}$ within some range.
- Different sampling methods.

Example:

The average lifetime of a light bulb is 3000 hours with a standard deviation of 696 hours. A simple random sample of 36 bulbs is taken.

(a) What are the expected value, standard deviation, and shape of the sampling distribution of $\bar{x}$?

(b) What is the probability that the average lifetime in the sample will be between 2670.56 and 2809.76 hours?

(c) What is the probability that the average lifetime in the sample will be greater than 3219.24 hours?

(d) What is the probability that the average lifetime in the sample will be less than 3180.96 hours?

[solution:]

(a) 

$$E(\bar{X}) = \mu = 3000 \quad \sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}} = \frac{696}{\sqrt{36}} = 116$$

bell shape like normal distribution.

(b) 

$$P\left(2670.56 \leq \bar{X} \leq 2809.76\right)$$

$$= P\left(\frac{2670.56 - 3000}{116} \leq \frac{\bar{X} - \mu}{\sigma_{\bar{X}}} = \frac{\bar{X} - 3000}{116} \leq \frac{2809.76 - 3000}{116}\right)$$

$$\approx P(-2.84 \leq Z \leq -1.64) = 0.0482$$
Example:

In a university, 10% of the students live in the dormitories. A random sample of 100 students is selected for a particular study.

(a) What is the probability that the sample proportion (the proportion living in the dormitories) is between 0.172 and 0.178?

(b) What is the probability that the sample proportion (the proportion living in the dormitories) is greater than 0.025?

[solution:]

\[ p = 0.1, \ n = 100, \ \sigma_p = \sqrt{\frac{p(1-p)}{n}} = \sqrt{\frac{0.1 \cdot (1-0.1)}{100}} = 0.03 \quad \text{(since)} \]

\[ np = 10 \geq 5, \ n(1-p) = 90 \geq 5 \]

(a)

\[ P(0.172 \leq \bar{p} \leq 0.178) = P \left( \frac{0.172 - 0.1}{0.03} \leq \frac{\bar{p} - p}{\sigma_p} = \frac{\bar{p} - 0.1}{0.03} \leq \frac{0.178 - 0.1}{0.03} \right) \approx P(2.4 \leq Z \leq 2.6) = 0.0035 \]

(b)

\[ P(0.025 \leq \bar{p}) = P \left( \frac{0.025 - 0.1}{0.03} \leq \frac{\bar{p} - p}{\sigma_p} = \frac{\bar{p} - 0.1}{0.03} \right) \approx P(-2.5 \leq Z) = 0.9938 \]
Chapter 8:

- Construct a \((1 - \alpha)\times100\%\) confidence intervals in large and small sample cases.
- Determine sample size based on the desired margin of error

**Example:**

Suppose we have the following data from a normal population

<table>
<thead>
<tr>
<th></th>
<th>50</th>
<th>48</th>
<th>55</th>
<th>52</th>
<th>53</th>
<th>46</th>
<th>54</th>
<th>50</th>
</tr>
</thead>
</table>

Provide a 95% confidence interval for the population mean.

[solution:]

\[
n = 8, \ \alpha = 0.05, \bar{x} = 51, s = 3.07. \ \text{Then, a 95\% confidence interval is}
\]

\[
\bar{x} \pm t_{n-1, \alpha/2} \frac{s}{\sqrt{n}} = \bar{x} \pm t_{7, 0.025} \frac{3.07}{\sqrt{8}} = 51 \pm 2.365 \frac{3.07}{\sqrt{8}} = 51 \pm 2.57 = [48.43, 53.57]
\]

**Example:**

100 students are asked whether they like or dislike statistics. The responses are given below:

<table>
<thead>
<tr>
<th>Response</th>
<th>disliked</th>
<th>liked</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency</td>
<td>60</td>
<td>40</td>
</tr>
</tbody>
</table>

(a) Construct a 95\% confidence interval for the proportion of the students who will not like statistics.

(b) With a 0.90 probability, how large of a sample needs to be taken to provide a margin of error of 5\% or less.

[solution:]
(a) \[ n = 100, \alpha = 0.05, \bar{p} = \frac{60}{100} = 0.6. \]

Then, a 95\% confidence interval is
\[
\bar{p} \pm z_{\alpha/2} \sqrt{\frac{\bar{p}(1 - \bar{p})}{n}} = \bar{p} \pm z_{0.025} \sqrt{\frac{\bar{p}(1 - \bar{p})}{n}} = 0.6 \pm 1.96 \sqrt{\frac{0.6 \cdot (1 - 0.6)}{100}} \\
= 0.6 \pm 0.096 = [0.504, 0.696]
\]

(b) \[ E = 0.05, \alpha = 0.1. \]

Then,
\[
n = \frac{z_{\alpha/2}^2 \bar{p}(1 - \bar{p})}{E^2} = \frac{z_{0.05}^2 \cdot 0.6 \cdot (1 - 0.6)}{(0.05)^2} = \frac{(1.645)^2 \cdot 0.6 \cdot (1 - 0.6)}{(0.05)^2} = 259.7784
\]

\[ \Rightarrow \quad n = 260. \]