Final 2021. 06. 22

1. (20%)

(a) The following data have been collected for a sample from a normal population

2	4	7	11

As $\alpha = 0.01$, test the hypothesis $H_0: \mu \geq 8$ $vs. H_a: \mu < 8$ by the ***classical (critical value) method***.

(b) A new brand of breakfast cereal is being market tested. The consumers were asked whether they liked or disliked the cereal. You are given their responses below. Let ***p*** be the proportion of all consumers who will like the cereal.

Response	Frequency
Liked	1488
Disliked	912
	2400

Test H_0:p=0.64 vs. H_a:p≠0.64 based on the ***p-value method*** at $\alpha = 0.05$.

2. (40%)

(a) Consider the following data for two random samples taken from two normal populations with equal variances. Consider the data as the ***independent-samples*** data.

Sample 1	11	12	8	7	7	9
Sample 2	5	8	6	7	4	

(i) Use t test for the following hypothesis

$$H_0: \mu_1 - \mu_2 \leq 3 \ \ vs. H_a: \mu > 3,$$

by ***the classical method (critical value)*** at $\alpha = 0.05$.

(ii) Test the following hypothesis

$$H_0: \mu_1 - \mu_2 = 0 \ \ vs. H_a: \mu_1 - \mu_2 \neq 0,$$

by ***the confidence interval method*** with $\alpha = 0.1$.

(b) The following results are sample proportions for two independent samples taken from the two populations.

Sample 1	Sample 2
$n_1 = 200$	$n_2 = 200$
$\overline{p}_1 = 0.2$	$\overline{p}_2 = 0.3$

(i) Find the 95% confidence interval of $p_1 - p_2$.

(ii) Test $H_0: p_2 \geq p_1$ $vs. H_a: p_2 < p_1$ at $\alpha = 0.02$ by ***the p-value method***.

3. (15%) The data were selected from each of three normal populations with equal variances. The data obtained follow.

Observation	Sample 1	Sample 2	Sample 3
1	8	6	8
2	5	6	7
3	3	7	9
4	6	5	6
5	6	3	5
Sample mean	5.6	5.4	7
Sample variance	3.3	2.3	2.5

Set up the ANOVA table for this problem and test the null hypothesis that the three population means are equal , i.e., $H_0: \mu_1 = \mu_2 = \mu_3$, at $\alpha = 0.05$.

4. (30%)

(a) A lottery is conducted that involves the random selection of numbers from 1 to 4. To make sure that the lottery is fair, a sample of 200 was taken. The following results were obtained:

Value	1	2	3	4
Frequency	40	55	60	45

Using chi-square statistic to test the hypothesis that the lottery is fair, i.e., $H_0: p_1 = p_2 = p_3 = p_4 = 0.25$ at $\alpha = 0.05$, where $p_i, i = 1, 2, 3, 4,$ is the probability of number i being selected.

(b) Shown below is a 2 x 2 contingency table with observed values from a sample of 100. At $\alpha = 0.1$, using chi-square statistic to test for independence of the row and column factors:

		Column Factor		
		X	Y	Total
Row Factor	A	25	35	60
	B	25	15	40
	Total	50	50	100

5. (30%) For the regression model $y_i = \beta_0 + \beta_1 x_i + \epsilon_i, i = 1, \cdots, 6, \epsilon_i \sim N(0, \sigma^2)$, with the following results:

$$\bar{x} = 10,\ \bar{y} = 15,\ \sum_{i=1}^{6}(x_i - \bar{x})^2 = 2500,$$

$$\sum_{i=1}^{6}(y_i - \bar{y})^2 = 6400,\ \sum_{i=1}^{6}(x_i - \bar{x})(y_i - \bar{y}) = 2000.$$

(a) Find the least squares estimate and the fitted regression equation.

(b) Provide an ANOVA table and use *F* statistic to test $H_0: \beta_1 = 0$ at $\alpha = 0.05$.

(c) Determine R^2 and r_{XY}.