

# Midterm

2021. 4. 20

1. (30%) A sample size of 900 provides a sample mean of 55 and sample standard deviation of 45.
- (a) Find the 90% confidence interval for the population mean.
- (b) With a 95% confidence interval of length 2, what sample size would be required to estimate the population mean?

2. (30%) The following data have been collected for a sample from a normal population

2	4	7	11
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- (a) Find the 95% confidence interval for the population mean.
- (b) With a 99% confidence level, what sample size would be required to estimate the population mean with margin error equal to 1.5?
3. (30%) A sample size of 2500 provided a sample proportion of  $\bar{p} = 0.2$ .
- (a) Find the 95% confidence interval for the population proportion.
- (b) With a 95% confidence interval, what sample size would be required to estimate the population proportion with margin error equal to 0.01?
4. (35%) A sample size of 225 provides a sample mean of 37 and sample standard deviation of 30.
- (a) (10%) As  $\alpha = 0.05$ , test the hypothesis
- $$H_0: \mu \leq 41 \text{ vs. } H_a: \mu > 41$$
- by the classical method, i.e., by using the critical value.
- (b) (10%) As  $\alpha = 0.01$ , test the hypothesis
- $$H_0: \mu \geq 34 \text{ vs. } H_a: \mu < 34$$
- by the classical method, i.e., by using the critical value.
- (c) (15%) As  $\alpha = 0.05$ , test the hypotheses in (a) and (b), i.e.,
- (i)  $H_0: \mu \leq 41 \text{ vs. } H_a: \mu > 41$
- (ii)  $H_0: \mu \geq 34 \text{ vs. } H_a: \mu < 34$
- by using the p-value method.

5. (10%) 2100 customers are asked whether they like or dislike some new drink. The responses are given below:

Response	Liked	Disliked
Number of Customers	c	2100 - c

If the length of a 95% confidence interval for the proportion p of all customers

who will like the new drink is 0.0392, find c.

## Equations:

### Confidence interval:

$$(\text{point estimate}) \pm [(z_{\alpha/2}, t_{n-1, \alpha/2}) * \\ (\text{standard error of point estimate})]$$

### Sample size for an interval estimate:

$$n = \frac{(z_{\alpha/2})^2 s^2}{E^2}, n = \frac{(z_{\alpha/2})^2 \bar{p}(1 - \bar{p})}{E^2}$$