Review 2

Chapter 9

Type I and II errors:

$$\alpha = P(H_0 \ is \ true \ but \ is \ rejected)$$
$$\beta = P(H_a \ is \ true \ but \ not \ reject \ H_0)$$

Hypothesis testing:

$$\text{test statistic} = \frac{point \ estimate - mean \ of \ point \ estimate \ under \ H_0}{standard \ deviation \ (error) \ of \ point \ estimate \ under \ H_0}$$

Summary table:

	Sample Mean μ		Sample Proportion p
	$n \geq 30$	$n < 30$, normal population	
Test Statistic	$z = \frac{\bar{x} - \mu_0}{\sigma_{\bar{X}}}$ or $\frac{\bar{x} - \mu_0}{s_{\bar{X}}}$	$t = \frac{\bar{x} - \mu_0}{s_{\bar{X}}}$	$z = \frac{\bar{p} - p_0}{\sigma_{\bar{P}}}$
Classical approach (critical values)	$-z_\alpha, z_\alpha, z_{\alpha/2}$	$-t_{n-1,\alpha}, t_{n-1,\alpha}, t_{n-1,\alpha/2}$	$-z_\alpha, z_\alpha, z_{\alpha/2}$
Distribution (p − value)	Z	$T(n-1)$	Z
C. I.	$\bar{x} \pm z_{\alpha/2}\sigma_{\bar{X}}$ or $\bar{x} \pm z_{\alpha/2}s_{\bar{X}}$	$\bar{x} \pm t_{n-1,\alpha/2}s_{\bar{X}}$	$\bar{p} \pm z_{\alpha/2}s_{\bar{P}}$

Example 1:

A sample size of 40 provides a sample mean of 16.5 and sample standard deviation of 7.

(a) As $\alpha = 0.1$, test the hypothesis $H_0: \mu \leq 15 \ vs \ H_a: \mu > 15$ based on the classical hypothesis test procedure.

(b) As $\alpha = 0.05$, test the hypothesis $H_0: \mu \leq 15 \ vs \ H_a: \mu > 15$ based on the p-value.

(c) As $\alpha = 0.06$, test the hypothesis $H_0: \mu = 15 \ vs \ H_a: \mu \neq 15$ based on the confidence interval.

[Solution:]

(a)

$$n = 40, \bar{x} = 16.5, s = 7, \mu_0 = 15, \alpha = 0.1.$$

Then,

$$z = \frac{\bar{x} - \mu_0}{\left(s/\sqrt{n}\right)} = \frac{16.5 - 15}{\left(7/\sqrt{40}\right)} = 1.355 > 1.28 = z_{0.1} = z_\alpha \Rightarrow reject \ H_0.$$

(b) $\alpha = 0.05$.

$$p - value = P(Z > z) = P(Z > 1.355) = 0.5 - 0.4131 = 0.0869 > 0.05,$$

we do not reject H_0.

(c) A $(1 - \alpha) \cdot 100\% = 94\%$ confidence interval for μ is

$$\bar{x} \pm z_{\alpha/2} \frac{s}{\sqrt{n}} = 16.5 \pm z_{0.03} \frac{7}{\sqrt{40}} = 16.5 \pm 1.88 \cdot 1.107 = [14.42, 18.58].$$

Since

$$\mu_0 = 15 \in [14.42, 18.58],$$

we do not reject H_0.

Example 2:

It is believed that the running time of movies is normally distributed with mean to 140 minutes (i.e., $H_0: \mu = 140 \ vs \ H_a: \mu \neq 140$). A sample of 4 movies was taken and the following running times were obtained,

150	150	180	170

At the 5% level of significance,

(a) test the hypothesis based on the classical hypothesis test procedure.

(b) using a p-value, test the hypothesis.

(c) using a confidence interval, test the hypothesis.

[Solution:]

$$n = 4, \bar{x} = 162.5, s = 15, \alpha = 0.05.$$

(a)

$$|t| = \left|\frac{\bar{x} - \mu_0}{\left(s/\sqrt{n}\right)}\right| = \left|\frac{162.5 - 140}{\left(15/\sqrt{4}\right)}\right| = 3 < 3.182 = t_{3,0.025} = t_{n-1,\alpha/2},$$

we do not reject H_0.

(b) Since

$$p - value = P(|T(3)| > 3) > P(|T(3)| > 3.182) = 0.05 = \alpha,$$

we do not reject H_0.

(c) A $(1 - \alpha) \cdot 100\% = 95\%$ confidence interval for μ is

$$\overline{x} \pm t_{n-1,\alpha/2}\frac{s}{\sqrt{n}} = 162.5 \pm t_{3,0.025} \cdot \frac{15}{\sqrt{4}} = [138.63, 186.36].$$

Since

$$\mu_0 = 140 \in [138.63, 186.36],$$

we do not reject $\mathrm{H_0}$.

Example 3:

A random sample of 400 people was taken. 228 of the people in the sample favored candidate A. We are interested in determining whether or not the proportion in favor of candidate A is significantly less than 50%.

(a) As $\alpha = 0.1$ and $H_0: p \leq 0.5$ vs $H_a: p > 0.5$, test the hypothesis based on the classical hypothesis test procedure.

(b) As $\alpha = 0.03$ and $H_0: p \leq 0.5$ vs $H_a: p > 0.5$, test the hypothesis based on the p-value.

(c) As $\alpha = 0.1$ and $H_0: p = 0.5$ vs $H_a: p \neq 0.5$, develop a confidence interval estimate to test the hypothesis.

[Solution:]

$$p_0 = 0.5, n = 400, \overline{p} = \frac{228}{400} = 0.57.$$

(a) $\alpha = 0.1$,

$$z = \frac{\overline{p} - p_0}{\left(\sqrt{\frac{p_0(1-p_0)}{n}}\right)} = \frac{0.57 - 0.5}{\left(\sqrt{\frac{0.5(1-0.5)}{400}}\right)} = 2.8 > 1.28 = z_{0.1} = z_\alpha,$$

we reject H_0.

(b)

$$p - value = P(Z > z) = P(Z > 2.8) = 0.0026 < 0.03 = \alpha,$$

we reject H_0.

(c) A $(1-\alpha) \cdot 100\% = 95\%$ confidence interval for p is

$$\overline{p} \pm z_{\alpha/2}\sqrt{\frac{\overline{p}(1-\overline{p})}{n}} = 0.57 \pm \left(1.645\sqrt{\frac{0.57(1-0.57)}{400}}\right) = [0.53, 0.61].$$

Since

$$p_0 = 0.5 \notin [0.53, 0.61],$$

we reject H_0.