Quiz 1 *2021. 03. 30*

1. (30%)

(a) (20%) A population proportion is 0.5. A simple random sample of size 2500 will be taken for (i), and the sample proportion $\bar{P}$ will be used to estimate the population proportion *p*.

(i) What is the probability that the sample proportion will be within ± 0.01 of the population proportion, i.e. $P(|\bar{P} - p| \leq 0.01)$?

(ii) Suppose the probability that a sample proportion will be within ± 0.005 of the population proportion is 0.95, i.e., $P(|\bar{P} - p| \leq 0.005) = 0.95$. What is the sample size *n*?

(b) (10%) Suppose we have a population of 72 elements, $y_1, y_2, \cdots, y_{72}$,

11	12	13	14	15	16	17	18	19
21	23	5	7	29	2	24	6	38
3	6	27	28	29	0	2	31	8
42	34	6	8	9	1	3	5	47
32	50	40	24	33	44	45	48	44
47	31	36	39	46	45	39	38	45
27	43	54	36	34	48	23	36	42
34	39	34	35	42	53	28	49	39

Suppose the first row of the table of random number is

13154 71744 59986 58683 51102 93108 80714 15141 63271 79945

(i) Please use *systematic sampling* to obtain a sample of *3* elements.

(ii) Consider the population as 3 stratums (in order), i..e,

Stratum 1:$y_1, \cdots, y_{24}$; Stratum 2:$y_{25}, \cdots, y_{48}$; Stratum 3: $y_{49}, \cdots, y_{72}$

Please use *stratified random sampling* to obtain a sample of 5 *data*, 1 *data* from stratum 1, 1 *data* from stratum 2, 3 *data* from stratum

2. (30%) A sample size of 1600 provides a sample mean of 51 and sample standard deviation of 20.

(a) Develop a 95% confidence interval for the population mean.

(b) Develop a 70% confidence interval for the population mean.

(c) If another sample with sample size $n \geq 30$ and sample deviation 30 provides a 95% confidence interval of which length is the half ($1/2$) of the length of the confidence interval given in (a), find n.

3. (20%) The following data have been collected for a sample from a normal population

2	**4**	**6**	**4**

(a) Find the 90% confidence interval for the population mean.

(b) With a 95% confidence interval , what size sample would be required to estimate the population mean with the margin error equal to 0.25?

4. (20%) A sample of 900 provided a sample proportion of $\bar{p} = 0.1$.

(a) Find the 95% confidence interval for the population proportion.

(b) With a 90% confidence level, what sample size would be required to estimate the population proportion with a confidence interval with the length equal to 0.02?

5. (20%)

(a) Let *X* be normal random variable with mean μ and variance 1. For the following hypothesis test $H_0: \mu = 3$ vs $H_a: \mu > 3$, i.e., $H_0: X \sim N(3,1)$ vs $H_a: X \sim N(\mu,1), \mu > 3$, we reject H_0 as $X > 4$. Please calculate α and find the range of β.

(b) Suppose that the sample size $n \geq 30$ and the population variance σ^2 is known. Please derive the $100(1-\alpha)\%$ confidence interval with the form $(-\infty, c]$ for the population mean, where c is the quantity to be determined. (Hint: using sampling distribution of the sample mean $\bar{X}$)