# *Review 1*

$(1-\alpha)\cdot 100\%$ confidence interval:

$(\text{point estimate}) \pm [(z_{\alpha/2}, t_{n-1,\alpha/2}) \cdot (\text{standard error of point estimate})]$

**Population mean** $\mu$:

- Large sample $n \geq 30$

$$\bar{x} \pm z_{\alpha/2}\sigma_{\bar{X}} = \bar{x} \pm z_{\alpha/2}\frac{\sigma}{\sqrt{n}} = \left[\bar{x} - z_{\alpha/2}\frac{\sigma}{\sqrt{n}}, \bar{x} + z_{\alpha/2}\frac{\sigma}{\sqrt{n}}\right]$$

or

$$\bar{x} \pm z_{\alpha/2}s_{\bar{X}} = \bar{x} \pm z_{\alpha/2}\frac{s}{\sqrt{n}} = \left[\bar{x} - z_{\alpha/2}\frac{s}{\sqrt{n}}, \bar{x} + z_{\alpha/2}\frac{s}{\sqrt{n}}\right]$$

- Small sample $n < 30$, normal population

$$\bar{x} \pm t_{n-1,\alpha/2}s_{\bar{X}} = \bar{x} \pm t_{n-1,\alpha/2}\frac{s}{\sqrt{n}} = \left[\bar{x} - t_{n-1,\alpha/2}\frac{s}{\sqrt{n}}, \bar{x} + t_{n-1,\alpha/2}\frac{s}{\sqrt{n}}\right]$$

**Population proportion** $p$:

$$\bar{p} \pm z_{\alpha/2}s_{\bar{P}} = \bar{p} \pm z_{\alpha/2}\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$$

$$= \left[\bar{p} - z_{\alpha/2}\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}, \bar{p} + z_{\alpha/2}\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}\right]$$

**Sample size for an interval estimate:**

$$n = \frac{z_{\alpha/2}^2 s^2}{E^2}, n = \frac{z_{\alpha/2}^2\bar{p}(1-\bar{p})}{E^2}.$$

**Chapter 9**

**Type I and II errors:**

$$\alpha = P(H_0 \text{ is true but is rejected})$$
$$\beta = P(H_a \text{ is true but not reject } H_0)$$

**Example 1:**

A sample of $625$ provides a sample mean of $30$ and a sample standard deviation of $20$.

(a) Develop a $80\%$ confidence interval for the population mean.

(b) With a $90\%$ confidence interval, what size sample would be required to estimate the population mean with margin error equal to $0.5$?

[Solution:]

(a) A $80\%$ confidence interval is

$$\bar{x} \pm z_{\alpha/2}\frac{s}{\sqrt{n}} = 30 \pm z_{0.1}\frac{20}{\sqrt{625}} = 30 \pm (1.28 \cdot 0.8) = [28.98, 31.02].$$

(b) $E = 0.5$.

$$n = \frac{z_{\alpha/2}^2 s^2}{E^2} = \frac{z_{0.05}^2 20^2}{(0.5)^2} = \frac{(1.645)^2 \cdot 400}{(0.5)^2} = 4329.64 \Rightarrow n = 4330.$$

**Example 2:**

You are given a random sample of $4$ observations from a normal population

| | | | |
|---|---|---|---|
| 80 | 72 | 88 | 72 |

(a) Find the $90\%$ confidence interval for the population mean.

(b) With a $95\%$ confidence interval of length $5$, what size sample would be required to estimate the population mean?

[Solution:]

(a)

$$n = 4, \alpha = 0.1,\ t_{n-1,\alpha/2} = t_{3,0.05} = 2.353, \bar{x} = 78, s = 7.66.$$

A $90\%$ confidence interval of $\mu$ is

$$\bar{x} \pm t_{n-1,\alpha/2}\frac{s}{\sqrt{n}} = 78 \pm t_{3,0.05}\frac{7.66}{\sqrt{4}} = 78 \pm (2.353 \cdot 3.83) = [68.99, 87.01].$$

(b) $E = {}^{5}/_{2} = 2.5, \alpha = 0.05$.

$$n = \frac{z_{\alpha/2}^2 s^2}{E^2} = \frac{z_{0.025}^2 \cdot (7.66)^2}{(2.5)^2} = \frac{(1.96)^2 \cdot 58.68}{6.25} = 36.07 \Rightarrow n = 37.$$

**Example 3:**

A random sample of $400$ people was taken. $228$ of the people in the sample favored candidate A.

(a) Develop a $90\%$ confidence interval estimate for the proportion in favor of candidate A.

(b) With a margin of error of $0.01$ or less at $95\%$ confidence, what size sample would be required to estimate the proportion in favor of candidate A?

[Solution:]

$$n = 400, \bar{p} = \frac{228}{400} = 0.57.$$

(a) A $90\%$ confidence interval for $p$ is

$$\overline{p} \pm z_{\alpha/2}\sqrt{\frac{\overline{p}(1-\overline{p})}{n}} = 0.57 \pm z_{0.05}\sqrt{\frac{0.57(1-0.57)}{400}}$$

$$= 0.57 \pm (1.645 \cdot 0.0248)$$

$$= [0.53, 0.61].$$

(b) $E = 0.01, \alpha = 0.05$.

$$n = \frac{z_{\alpha/2}^2 \overline{p}(1-\overline{p})}{E^2} = \frac{z_{0.025}^2 \cdot 0.57 \cdot (1-0.57)}{(0.01)^2} = 9415.76 \Rightarrow n = 9416.$$

**Example 4:**

For the following hypothesis test

$$H_0: X \sim binomial(3, {}^1/_3) \ \ vs \ \ H_a: X \sim Poisson(1).$$

where $binomial(3, {}^1/_3)$ has the distribution function

$$f_X(x) = \binom{3}{x}\left(\frac{1}{3}\right)^x\left(\frac{1}{3}\right)^{3-x}, x = 0, 1, 2, 3.$$

and $Poisson(1)$ has the distribution function

$$f_X(x) = \frac{e^{-1}}{x!}, x = 0, 1, 2, \cdots$$

we reject $H_0$ as $X \leq 2$. Please calculate $\alpha$ and $\beta$.

[Solution:]

$$\alpha = P(H_0 \ is \ true \ but \ is \ rejected) = P\left(X \sim binomial(3, {}^1/_3), X \leq 2\right)$$

$$= 1 - P\left(X \sim binomial(3, {}^1/_3), X = 3\right) = 1 - \binom{3}{3}\left(\frac{1}{3}\right)^3\left(\frac{1}{3}\right)^{3-3}$$

$$= \frac{26}{27}$$

and

$$\beta = P(H_a \ is \ true \ but \ not \ reject \ H_0) = P(X \sim Poisson(1), X > 2)$$

$$= 1 - P(X \sim Poisson(1), X \leq 2) = 1 - \left(\frac{e^{-1}}{0!} + \frac{e^{-1}}{1!} + \frac{e^{-1}}{2!}\right)$$

$$= 1 - 2.5e^{-1}$$