CHAPTER 13

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The Expected Return–Beta Relationship

Recall that if the expected return–beta relationship holds with respect to an observable ex ante efficient index, $M$, the expected rate of return on any security $i$ is

$$E(r_i) = r_f + \beta_i[E(r_M) - r_f]$$  (13.1)

where $\beta_i$ is defined as $\frac{\text{Cov}(r_i, r_M)}{\sigma^2_M}$. 
The Expected Return–Beta Relationship

- This is the most commonly tested implication of the CAPM. Early simple tests followed three basic steps: establishing sample data, estimating the SCL (security characteristic line), and estimating the SML (security market line).
Setting Up the Sample Data

- Determine a sample period of, for example, 60 monthly holding periods (5 years). For each of the 60 holding periods, collect the rates of return on 100 stocks, a market portfolio proxy (e.g., the S&P 500), and 1-month (risk-free) T-bills.
Your data thus consist of

- $R_{it}$ Returns on the 100 stocks over the 60-month sample period; $i = 1, \ldots, 100$, and $t = 1, \ldots, 60$.
- $R_{Mt}$ Returns on the S&P 500 index over the sample period.
- $r_{ft}$ Risk-free rate each month.

This constitutes a table of $102 \times 60 = 6,120$ rates of return.
Estimating the SCL

- View equation 13.1 as a security characteristic line (SCL), as in Chapter 10. For each stock, $i$, you estimate the beta coefficient as the slope of a \textit{first-pass regression} equation.
Estimating the SCL

- The terminology *first-pass* regression is due to the fact that the estimated coefficients will be used as input into a *second-pass* regression.

\[ r_{it} - r_{ft} = a_i + b_i(r_{Mt} - r_{ft}) + e_{it} \]
Estimating the SCL

You will use the following statistics in later analysis:

- $r_i - r_f$ Sample averages (over the 60 observations) of the excess return on each of the 100 stocks.
- $b_i$ Sample estimates of the beta coefficients of each of the 100 stocks.
- $r_M - r_f$ Sample average of the excess return of the market index.
- $\sigma^2(e_i)$ Estimates of the variance of the residuals for each of the 100 stocks.
Estimating the SCL

The sample average excess returns on each stock and the market portfolio are taken as estimates of expected excess returns, and the values of $b_i$ are estimates of the true beta coefficients for the 100 stocks during the sample period. The $\sigma^2(e_i)$ estimates the nonsystematic risk of each of the 100 stocks.
Estimating the SML

Now view equation 13.1 as a security market line (SML) with 100 observations for the stocks in your sample. You can estimate $\gamma_0$ and $\gamma_1$ in the following second-pass regression equation with the estimates $b_i$ from the first pass as the independent variable:

$$r_i - r_f = \gamma_0 + \gamma_1 b_i \quad (13.2)$$
Estimating the SML

- Compare equations 13.1 and 13.2; you should conclude that if the CAPM is valid, then $\gamma_0$ and $\gamma_1$ should satisfy

$$\gamma_0 = 0 \text{ and } \gamma_1 = \frac{r_M - r_f}{s}$$
Estimating the SML

- In fact, you can further argue that the key property of the expected return–beta relationship described by the SML is that the expected excess return on securities is determined only by the systematic risk (as measured by beta) and should be independent of the nonsystematic risk, as measured by the variance of the residuals, $\sigma^2(e_i)$, which also were estimated from the first-pass regression.
Estimating the SML

These estimates can be added as a variable in equation 13.2 of an expanded SML that now looks like this:

$$r_i - r_f = \gamma_0 + \gamma_1 b_i + \gamma_2 \sigma^2(e_i) \quad (13.3)$$

This second-pass regression is estimated with the hypothesis

$$\gamma_0 = 0; \quad \gamma_1 = \frac{r_M - r_f}{r_f}; \quad \gamma_2 = 0$$
Estimating the SML

- The hypothesis that $\gamma_2 = 0$ is consistent with the notion that nonsystematic risk should not be “priced,” that is, that there is no risk premium earned for bearing nonsystematic risk.
More generally, according to the CAPM, the risk premium depends only on beta. Therefore, any additional right-hand-side variable in equation 13.3 beyond beta should have a coefficient that is insignificantly different from zero in the second-pass regression.
Tests of the CAPM

- Early tests of the CAPM performed by John Lintner, and later replicated by Merton Miller and Myron Scholes, used annual data on 631 NYSE stocks for 10 years, 1954 to 1963, and produced the following estimates (with returns expressed as decimals rather than percentages): (next slide)
Tests of the CAPM

Coefficient: $\gamma_0 = 0.127 \quad \gamma_1 = 0.042 \quad \gamma_2 = 0.310$

Std error: 0.006 0.006 0.026

Sample average: $r_M - r_f = 0.165$
These results are inconsistent with the CAPM. First, the estimated SML is “too flat;” that is, the $\gamma_1$ coefficient is too small. The slope should be $r_M - r_f = .165$ (16.5% per year), but it is estimated at only .042.
The difference, .122, is about 20 times the standard error of the estimated, .006, which means that the measured slope of the SML is less than it should be by a statistically significant margin. At the same time, the intercept of the estimated SML, $\gamma_0$, which is hypothesized to be zero, in fact equals .127, which is more than 20 times its standard error of .006.
Tests of the CAPM

- The two-stage procedure employed by these researchers (i.e., first estimate security beats using a time-series regression and then use those betas to test the SML relationship between risk and average return) seems straightforward, and the rejection of the CAPM using this approach is disappointing. However, it turns out that there are several difficulties with this approach.
First and foremost, stock returns are extremely volatile, which lessens the precision of any tests of average return.

- For example, the average standard deviation of annual returns of the stocks in the S&P 500 is about 40%; the average standard deviation of annual returns of the stocks included in these tests is probably even higher.
Tests of the CAPM

- In addition, there are fundamental concerns about the validity of the tests.
  - First, the market index used in the tests is surely not the “market portfolio” of the CAPM.
Tests of the CAPM

- Second, in light of asset volatility, the security betas from the first-stage regressions are necessarily estimated with substantial sampling error and therefore cannot readily be used as inputs to the second-stage regression.
- Finally, investors cannot borrow at the risk-free rate, as assumed by the simple version of the CAPM. Let us investigate the implications of these problems in turn.
In what has become known as Roll’s critique, Richard Roll pointed out that:

1. There is a single testable hypothesis associated with the CAPM: The market portfolio is mean-variance efficient.
2. All the other implications of the model, the best-known being the linear relation between expected return and beta, follow from the market portfolio’s efficiency and therefore are not independently testable. There is an “if and only if” relation between the expected return-beta relationship and the efficiency of the market portfolio.
3. In any sample of observations of individual returns there will be an infinite number of ex post (i.e., after the fact) mean-variance efficient portfolios using the sample-period returns and covariances (as opposed to the ex ante expected returns and covariances). Sample betas calculated between each such portfolio and individual assets will be exactly linearly related to sample average returns.
In other words, if betas are calculated against such portfolios, they will satisfy the SML relation exactly whether or not the true market portfolio is mean-variance efficient in an ex ante sense.
The Market Index

4. The CAPM is not testable unless we know the exact composition of the true market portfolio and use it in the tests. This implies that the theory is not testable unless all individual assets are included in the sample.
5. Using a proxy such as the S&P 500 for the market portfolio is subject to two difficulties. First, the proxy itself might be mean-variance efficient even when the true market portfolio is not. Conversely, the proxy may turn out to be inefficient, but obviously this alone implies nothing about the true market portfolio’s efficiency.
The Market Index

Furthermore, most reasonable market proxies will be very highly correlated with each other and with the true market portfolio whether or not they are mean-variance efficient. Such a high degree of correlation will make it seem that the exact composition of the market portfolio is unimportant, whereas the use of different proxies can lead to quite different conclusions.
The Market Index

This problem is referred to as **benchmark error**, because it refers to the use of an incorrect benchmark (market proxy) portfolio in the tests of the theory.
Roll and Ross and Kandel and Stambaugh expanded Roll’s critique. Essentially, they argued that tests that reject a positive relationship between average return and beta point to inefficiency of the market proxy used in those tests, rather than refuting the theoretical expected return–beta relationship.
The Market Index

- Their work demonstrates that it is plausible that even highly diversified portfolios, such as the value- or equally weighted portfolios of all stocks in the sample, will fail to produce a significant average return–beta relationship.
The Market Index

- Roll and Ross (RR) derived an analytical characterization of market indexes (proxies for the market portfolio) that produce an arbitrary cross-sectional slope coefficient in the regression of average asset returns on beta. Their derivation applies to any universe of assets and requires only that the market proxy be constructed from that universe or one of its subsets.
The Market Index

- RR show that the set of indexes that produces a zero second-pass slope lies within a parabola that is tangent to the efficient frontier at the point corresponding to the global minimum variance portfolio.
Figure 13.1 shows one such configuration. In this plausible universe, where “plausible” is taken to mean that the return distribution is not extraordinary, the set of portfolios with zero slope coefficient in the return-beta regression lies near the efficient frontier. Thus even portfolios that are “nearly efficient” do not necessarily support the expected return–beta relationship.
Figure 13.1 Market index proxies that produce betas having no relation to expected returns

These proxies are located within a restricted region of the mean-variance space, a region bounded by a parabola that lies inside the efficient frontier except for a tangency at the global minimum variance point. The market proxy is located on the boundary at a distance of $M = 22$ basis points below the efficient frontier. While betas against this market proxy have zero cross-sectional correlation with expected returns, a market proxy on the efficient frontier just 22 basis points above it would produce betas that are perfectly positively collinear with expected returns.

RR concluded that the slope coefficient in the average return–beta regression cannot be relied on to test the theoretical expected return–beta relationship. It can indicate only that the market proxy that produces this result is inefficient in the second-pass regression.
The Market Index

- Many studies use the more sophisticated regression procedure called generalized least squares (GLS) to improve on statistical reliability. Can the use of GLS overcome the problems raised by Roll and Ross?
Kandel and Stambaugh (KS) extended this analysis and considered whether the use of generalized least squares regressions can overcome some of the problems identified by Roll and Ross. They found that GLS does help, but only to the extent that the researcher can obtain a nearly efficient market index.
KS considered the properties of the usual two-pass test of the CAPM in an environment in which borrowing is restricted but the zero-beta version of the CAPM holds.
The Market Index

- In this case, you will recall that the expected return–beta relationship describes the expected returns on a stock, a portfolio $E$ on the efficient frontier, and that portfolio’s zero-beta companion, $Z$ (see equation 9.9):

$$E(r_i) - E(r_Z) = \beta_i [E(r_E) - E(r_Z)] \quad (13.4)$$

where $\beta_i$ denotes the beta of security $i$ on efficient portfolio $E$. 
We cannot construct or observe the efficient portfolio $E$ (since we do not know expected returns and covariances of all assets), and so we cannot estimate equation 13.4 directly.
KS asked what would happen if we followed the common procedure of using a market proxy portfolio $M$ in place of $E$, and used as well the more efficient GLS egression procedure in estimating the second-pass regression for the zero-beta version of the CAPM, that is,

$$r_i - r_Z = \gamma_0 + \gamma_1 \times (\text{Estimated } \beta_i)$$
They showed that the estimated values of $\gamma_0$ and $\gamma_1$ will be biased by a term proportional to the relative efficiency of the market proxy. If the market index used in the regression is fully efficient, the test will be well specified. But the second-pass regression will provide a poor test of the CAPM if the proxy for the market portfolio is not efficient.
The Market Index

Thus, while GLS regressions may not give totally arbitrary results, as RR demonstrate may occur using standard OLS regressions, we still cannot test the model in a meaningful way without a reasonably efficient market proxy. Unfortunately, it is difficult to tell how efficient our market index is relative to the theoretical true market portfolio, so we cannot tell how good our tests are.
Measurement Error in Beta

- Roll’s critique tells us that CAPM tests are handicapped from the outset. But suppose that we could get past Roll’s problem by obtaining data on the returns of the true market portfolio. We still would have to deal with the statistical problems caused by measurement error in the estimates of beta from the first-stage regressions.
Measurement Error in Beta

- It is well known in statistics that if the right-hand-side variable of a regression equation is measured with error (in our case, beta is measured with error and is the right-hand-side variable in the second-pass regression), then the slope coefficient of the regression equation will be biased downward and the intercept biased upward.
Measurement Error in Beta

- This is consistent with the findings cited above, which found that the estimate of $\gamma_0$ was higher than predicted by the CAPM and that the estimate of $\gamma_1$ was lower than predicted.
Measurement Error in Beta

- Indeed, a well-controlled simulation test by Miller and Scholes confirms these arguments. In this test a random-number generator simulated rates of return with covariances similar to observed ones. The average returns were made to agree exactly with the CAPM expected return–beta relationship.
Measurement Error in Beta

- Miller and Scholes then used these randomly generated rates of return in the tests we have described as if they were observed from a sample of stock returns.
Measurement Error in Beta

- The results of this “simulated” test were virtually identical to those reached using real data, despite the fact that the simulated returns were constructed to obey the SML, that is, the true $\gamma$ coefficients were $\gamma_0 = 0$, $\gamma_1 = r_M - r_f$, and $\gamma_2 = 0$. 
Measurement Error in Beta

- This postmortem of the early test gets us back to square one. We can explain away the disappointing test results, but we have no positive results to support the CAPM-APT implications.
Measurement Error in Beta

- The next wave of tests was designed to overcome the measurement error problem that led to biased estimates of the SML. The innovation in these tests, pioneered by Black, Jensen, and Scholes (BJS), was to use portfolios rather than individual securities.
Measurement Error in Beta

- Combining securities into portfolios diversifies away most of the firm-specific part of returns, thereby enhancing the precision of the estimates of beta and the expected rate of return of the portfolio of securities. This mitigates the statistical problems that arise from measurement error in the beta estimates.
Measurement Error in Beta

- Obviously, however, combining stocks into portfolios reduces the number of observations left for the second-pass regression.
Measurement Error in Beta

- For example, suppose that we group our sample of 100 stocks into five portfolios of 20 stocks each. If the assumption of a single-factor market is reasonably accurate, then the residuals of the 20 stocks in each portfolio will be practically uncorrelated and, hence, the variance of the portfolio residual will be about one-twentieth the residual variance of the average stock.
Measurement Error in Beta

Thus the portfolio beta in the first-pass regression will be estimated with far better accuracy. However, now consider the second-pass regression. With individual securities we had 100 observations to estimate the second-pass coefficients. With portfolios of 20 stocks each we are left with only five observations for the second-pass regression.
Measurement Error in Beta

- To get the best of this trade-off, we need to construct portfolios with the largest possible dispersion of beta coefficients. Other things being equal, a sample yields more accurate regression estimates the more widely spaced are the observations of the independent variables.
Measurement Error in Beta

- Consider the first-pass regressions where we estimate the SCL, that is, the relationship between the excess return on each stock and the market’s excess return. If we have a sample with a great dispersion of market returns, we have a greater chance of accurately estimating the effect of a change in the market return on the return of the stock.
Measurement Error in Beta

- In our case, however, we have no control over the range of the market returns. But we can control the range of the independent variable of the second-pass regression, the portfolio betas. Rather than allocate 20 stocks to each portfolio randomly, we can rank portfolios by betas.
Measurement Error in Beta

- Portfolio 1 will include the 20 highest-beta stocks and Portfolio 5 the 20 lowest-beta stocks. In that case a set of portfolios with small nonsystematic components, \( e_p \), and widely spaced betas will yield reasonably powerful tests of the SML.
Measurement Error in Beta

- Fama and MacBeth used this methodology to verify that the observed relationship between average excess returns and beta is indeed linear and that nonsystematic risk does not explain average excess returns.
Measurement Error in Beta

- Using 20 portfolios constructed according to the BJS methodology. Fama and MacBeth expanded the estimation of the SML equation to include the square of the beta coefficient (to test for linearity of the relationship between returns and betas) and the estimated standard deviation of the residual (to test for the explanatory power of nonsystematic risk).
For a sequence of many subperiods they estimated for each subperiod, the equation

\[ r_i = \gamma_0 + \gamma_1 \beta_i + \gamma_2 \beta_i^2 + \gamma_3 \sigma(e_i) \]  

(13.5)
Measurement Error in Beta

- The term $\gamma_2$ measures potential nonlinearity of return, and $\gamma_3$ measures the explanatory power of nonsystematic risk, $\sigma(e_i)$. According to the CAPM, both $\gamma_2$ and $\gamma_3$ should have coefficients of zero in the second-pass regression.
Fama and MacBeth estimated equation 13.5 for every month of the period January 1935 through June 1968.

The results are summarized in Table 13.1, which shows average coefficients and $t$-statistics for the overall period as well as for three subperiods.
Table 13.1  Summary of Fama and MacBeth (1973) Study (all rates in basis points per month)

<table>
<thead>
<tr>
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</tr>
</thead>
<tbody>
<tr>
<td>Av. $r_t$</td>
<td>13</td>
<td>2</td>
<td>9</td>
<td>26</td>
</tr>
<tr>
<td>Av. $y_{0} - r_t$</td>
<td>8</td>
<td>10</td>
<td>8</td>
<td>5</td>
</tr>
<tr>
<td>Av. $t(y_{0} - r_t)$</td>
<td>0.20</td>
<td>0.11</td>
<td>0.20</td>
<td>0.10</td>
</tr>
<tr>
<td>Av. $r_M - r_t$</td>
<td>130</td>
<td>195</td>
<td>103</td>
<td>95</td>
</tr>
<tr>
<td>Av. $y_1$</td>
<td>114</td>
<td>118</td>
<td>209</td>
<td>34</td>
</tr>
<tr>
<td>Av. $t(y_1)$</td>
<td>1.85</td>
<td>0.94</td>
<td>2.39</td>
<td>0.34</td>
</tr>
<tr>
<td>Av. $y_2$</td>
<td>-26</td>
<td>-9</td>
<td>-76</td>
<td>0</td>
</tr>
<tr>
<td>Av. $t(y_2)$</td>
<td>-0.86</td>
<td>-0.14</td>
<td>-2.16</td>
<td>0</td>
</tr>
<tr>
<td>Av. $y_3$</td>
<td>516</td>
<td>817</td>
<td>-378</td>
<td>960</td>
</tr>
<tr>
<td>Av. $t(y_3)$</td>
<td>1.11</td>
<td>0.94</td>
<td>-0.67</td>
<td>1.11</td>
</tr>
<tr>
<td>Av. $R - SQR$</td>
<td>0.31</td>
<td>0.31</td>
<td>0.32</td>
<td>0.29</td>
</tr>
</tbody>
</table>
Fama and MacBeth observed that the coefficients on residual standard deviation (nonsystematic risk), denoted by $\gamma_3$, fluctuate greatly from month to month and were insignificant, consistent with the hypothesis that nonsystematic risk is not rewarded by higher average returns.
Likewise, the coefficients on the square of beta, denoted by $\gamma_2$, were insignificant, consistent with the hypothesis that the expected return–beta relationship is linear.
Measurement Error in Beta

- With respect to the expected return-beta relationship, however, the picture is mixed. The estimated SML is too flat, consistent with previous studies, as can be seen from the fact that $\gamma_1 - r_f$ is positive, and that $\gamma_1$ is, on average, less than $r_M - r_f$. On the positive side, the difference does not appear to be significant, so that the CAPM is not clearly rejected.
In conclusion, these tests of the CAPM provide mixed evidence on the validity of the theory. We can summarize the results as follows:

- The insights that are supported by the single-factor CAPM and APT are as follows:
  - Expected rates of return are linear and increase with beta, the measure of systematic risk.
  - Expected rates of return are not affected by nonsystematic risk.
Measurement Error in Beta

- The single-variable expected return–beta relationship predicted by either the risk-free rate or the zero-beta version of the CAPM is not fully consistent with empirical observation.
Thus, although the CAPM seems qualitatively correct in that $\beta$ matters and $\sigma(e_i)$ does not, empirical tests do not validate its quantitative predictions.
The EMH and the CAPM

- Roll’s critique also provides a positive avenue to view the empirical content of the CAPM and APT. Recall, as Roll pointed out, that the CAPM and the expected return-beta relationship follow directly from the efficiency of the market portfolio. This means that if we can establish that the market portfolio is efficient, we would have no need to further test the expected return–beta relationship.
The EMH and the CAPM

As demonstrated in Chapter 12 on the efficient market hypothesis, proxies for the market portfolio such as the S&P 500 and NYSE index have proven hard to beat by professional investors. This is perhaps the strongest evidence for the empirical content of the CAPM and APT.
Accounting for Human Capital and Cyclical Variations in Asset Betas

- We are reminded of two important deficiencies of the tests of the single-index models:
  - Only a fraction of the value of assets in the United States is traded in capital markets; perhaps the most important nontraded asset is human capital.
  - There is ample evidence that asset betas are cyclical and that accounting for this cyclicality may improve the predictive power of the CAPM.
Accounting for Human Capital and Cyclical Variations in Asset Betas

- One of the CAPM assumptions is that all assets are traded and accessible to all investors. Mayers proposed a version of CAPM that accounts for a violation of this assumption; this requires an additional term in the expected return–beta relationship.
An important nontraded asset that may partly account for the deficiency of standard market proxies such as the S&P 500 is human capital. The value of future wages and compensation for expert services is a significant component of the wealth of investors who expect years of productive careers prior to retirement.
Moreover, it is reasonable to expect that changes in human capital are far less than perfectly correlated with asset returns, and hence they diversify the risk of investor portfolios.
Jaganathan and Wang (JW) used a proxy for changes in the value of human capital based on the rate of change in aggregate labor income. In addition to the standard security betas estimated using the value-weighted stock market index, which we denote $\beta_{vw}$, JW also estimated the betas of assets with respect to labor income growth, which we denote $\beta_{labor}$. 
Finally, JW considered the possibility that business cycles affect asset betas, an issue that has been examined in a number of other studies. They used the difference between the yields on low- and high-grade corporate bonds as a proxy for the state of the business cycle and estimate asset betas relative to this business cycle variable; we denote this beta as $\beta_{\text{prem}}$. 
Accounting for Human Capital and Cyclical Variations in Asset Betas

With the estimates of these three betas for several stock portfolios, JW estimated a second-pass regression which includes firm size (market value of equity, denoted ME):

\[ E(R_i) = c_0 + c_{\text{size}} \log(\text{ME}) + c_{\text{vw}} \beta_{\text{vw}} + c_{\text{prem}} \beta_{\text{prem}} + c_{\text{labor}} \beta_{\text{labor}} \]

(13.6)
Their results are far more supportive of the CAPM than earlier tests. The explanatory power of the equations that include JW’s expanded set of explanatory variables (which they call a “conditional” CAPM because beta is conditional on the state of the economy) is much greater than in earlier test, and the significance of the size variable disappears.
Figure 13.2 shows the improvements of these tests more dramatically.

- Figure 13.2A shows that the conventional CAPM indeed works poorly. The figure compares predicted security returns fitted using the firm’s beta versus actual returns. There is obviously almost no relationship between the two. This is indicative of the weak performance of the conventional CAPM in empirical tests.
Accounting for Human Capital and Cyclical Variations in Asset Betas

- But if we use the conditional CAPM to compare fitted to actual returns, as in Figure 13.2B, we get a dramatically improved fit. Moreover, adding firm size to this model turns out to do nothing to improve the fit. Moreover, adding firm size to this model turns out to do nothing to improve the fit.

- We conclude that firm size does not improve return predictions once we account for the variables addressed in the conditional CAPM.
Figure 13.2 Fitted expected returns versus realized average returns

Each scatter point in the graph represents a portfolio, with the realized average return as the horizontal axis and the fitted expected return as the vertical axis. For each portfolio $i$, the realized average return is the time-series average of the portfolio return, and the fitted expected return is the fitted value for the expected return, $E(R_i)$, in the following regression model:

$$E(R_i) = c_0 + c_{\beta \text{w}} \beta_i^{\text{w}}$$

where $\beta_i^{\text{w}}$ is the slope coefficient in the OLS regression of the portfolio return on a constant and the return on the value-weighted index portfolio of stocks. The straight line in the graph is the 45° line from the origin.

Each scatter point in the graph represents a portfolio, with the realized average return as the horizontal axis and the fitted expected return as the vertical axis. For each portfolio $i$, the realized average return is the time-series average of the portfolio return, and the fitted expected return is the fitted value for the expected return, $E(R_i)$, in the following regression model:

$$E(R_i) = c_0 + c_{\beta \text{w}} \beta_i^{\text{w}} + c_{\beta \text{b}} \beta_i^{\text{b}} + c_{\beta \text{g}} \beta_i^{\text{g}}$$

where $\beta_i^{\text{w}}$ is the slope coefficient in the OLS regression of the portfolio return on a constant and the return on the value-weighted index portfolio of stocks, $\beta_i^{\text{b}}$ is the slope coefficient in the OLS regression of the portfolio return on a constant and the yield spread between low- and high-grade corporate bonds, and $\beta_i^{\text{g}}$ is the slope coefficient in the OLS regression of the portfolio return on a constant and the growth rate in per capita labor income. The straight line in the graph is the 45° line from the origin.
Accounting for Human Capital and Cyclical Variations in Asset Betas

- JW also compare the conditional CAPM to the Fama and French 3-factor model and find that the significance of the book-to-market and size factors disappears once we account for human capital and cyclical variation of the single-index betas.
The multifactor CAPM and APT are elegant theories of how exposure to systematic risk factors should influence expected returns, but they provide little guidance concerning which factors (sources of risk) ought to result in risk premiums.
A test of this hypothesis would require three stages:

2. Identification of portfolios that hedge these fundamental risk factors.
3. Test of the explanatory power and risk premiums of the hedge portfolios.
Chen, Roll and Ross, identify several possible variables that might proxy for systematic factors:

- IP = Growth rate in industrial production
- EI = Changes in expected inflation measured by changes in short-term (T-bill) interest rates
- UI = Unexpected inflation defined as the difference between actual and expected inflation
A Macro Factor Model

- CG = Unexpected changes in risk premiums measured by the difference between the returns on corporate Baa-rated bonds and long-term government bonds
- GB = Unexpected changes in the term premium measured by the difference between the returns on long- and short-term government bonds
- M = Stock market index
A Macro Factor Model

- With the identification of these potential economic factors, Chen, Roll, and Ross skipped the procedure of identifying factor portfolios (the portfolios that have the highest correlation with the factors).
A Macro Factor Model

Instead, by using the factors themselves, Chen, Roll, and Ross implicitly assumed that factor portfolios exist that can proxy for the factors. They use these factors in a test similar to that of Fama and MacBeth.
A Macro Factor Model

A critical part of the methodology is the grouping of stocks into portfolios. Recall that in the single-factor tests, portfolios were constructed to span a wide range of betas to enhance the power of the test. In a multifactor framework the efficient criterion for grouping is less obvious.
A Macro Factor Model

- Chen, Roll, and Ross chose to group the sample stocks into 20 portfolios by size (market value of outstanding equity), a variable that is known to be associated with average stock returns.
A Macro Factor Model

- They first used 5 years of monthly data to estimate the factor betas of the 20 portfolios in a first-pass regression. This is accomplished by estimating the following regressions for each portfolio:

\[ r = \alpha + \beta_M r_M + \beta_{IP} IP + \beta_{EI} EI + \beta_{UI} UI + \beta_{CG} CG + \beta_{GB} GB + \epsilon \]  

(13.7a)
A Macro Factor Model

- Chen, Roll, and Ross used as the market index both the value-weighted NYSE index (VWNY) and the equally weighted NYSE index (EWNY).
A Macro Factor Model

- Using the 20 sets of first-pass estimates of factor betas as the independent variables, Chen, Roll, and Ross now estimated the second-pass regression (with 20 observations, one for each portfolio):

\[ r = \gamma_0 + \gamma_M \beta_M + \gamma_{IP} \beta_{IP} + \gamma_{EI} \beta_{EI} + \gamma_{UI} \beta_{UI} + \gamma_{CG} \beta_{CG} + \gamma_{GB} \beta_{GB} + e \]  

(13.7b)
A Macro Factor Model

- In equation 13.7b the gammas become estimates of the risk premiums on the factors.
- Chen, Roll, and Ross ran this second-pass regression for every month of their sample period, reestimating the first-pass factor betas once every 12 months. They ran the second-pass tests in four variations.
<table>
<thead>
<tr>
<th></th>
<th>YP</th>
<th>IP</th>
<th>EI</th>
<th>UI</th>
<th>CG</th>
<th>GB</th>
<th>Constant</th>
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</thead>
<tbody>
<tr>
<td>A</td>
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<td>-0.672</td>
<td>7.941</td>
<td>-5.8</td>
<td>4.112</td>
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<tr>
<td></td>
<td>(0.538)</td>
<td>(3.727)</td>
<td>(-1.499)</td>
<td>(-2.052)</td>
<td>(2.807)</td>
<td>(-1.844)</td>
<td>(1.334)</td>
</tr>
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<td>B</td>
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<td>-0.125</td>
<td>-6.29</td>
<td>7.205</td>
<td>-5.211</td>
<td>4.124</td>
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<tr>
<td></td>
<td>(3.561)</td>
<td>(-1.640)</td>
<td>(-1.979)</td>
<td>(2.590)</td>
<td>(-1.690)</td>
<td>(1.361)</td>
<td></td>
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<tr>
<td>C</td>
<td>5.021</td>
<td>14.009</td>
<td>-0.128</td>
<td>-0.848</td>
<td>0.130</td>
<td>-5.017</td>
<td>6.409</td>
</tr>
<tr>
<td></td>
<td>(1.218)</td>
<td>(3.774)</td>
<td>(-1.666)</td>
<td>(-2.541)</td>
<td>(2.855)</td>
<td>(-1.576)</td>
<td>(1.848)</td>
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<td>D</td>
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<tr>
<td></td>
<td>(-0.633)</td>
<td>(3.054)</td>
<td>(-1.600)</td>
<td>(-2.376)</td>
<td>(2.972)</td>
<td>(-1.879)</td>
<td>(2.755)</td>
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</tbody>
</table>

VWNY = Return on the value-weighted NYSE index; EWNY = Return on the equally weighted NYSE index; IP = Monthly growth rate in industrial production; EI = Change in expected inflation; UI = Unanticipated inflation; CG = Unanticipated change in the risk premium (Baa and under return – long-term government bond return); GB = Unanticipated change in the term structure (long-term government bond return – Treasury-bill rate); and YP = Yearly growth rate in industrial production. Note that t-statistics are in parentheses.

A Macro Factor Model

- First (Table 13.3, parts A and B), they excluded the market index altogether and used two alternative measures of industrial production (YP based on annual growth of industrial production and MP based on monthly growth).
Finding that MP is more effective measure, they next included the two versions of the market index, EWNY and VWNY, one at a time (Table 13.3, parts C and D). The estimated risk premiums (the values for the parameters, $\gamma$) were averaged over all the second-pass regressions.
A Macro Factor Model

- Note in Table 13.3, parts C and D, that the two market indexes EWNY (equally weighted index of NYSE) and VWNY (the value-weighted NYSE index) are not statistically significant (their $t$-statistics of 1.218 and -.633 are less than 2). Note also that the VWNY factor has the “wrong” sign in that it seems to imply a negative market-risk premium.
A Macro Factor Model

- Industrial production (MP), the risk premium on corporate bonds (CG), and unanticipated inflation (UI) are the factors that appear to have significant explanatory power.
A Macro Factor Model

- These results indicate that it may be possible to hedge some economic factors that affect future consumption risk with appropriate portfolios. A CAPM or APT multifactor equilibrium expected return–beta relationship may one day supersede the now widely used single-factor model.
The multifactor model that occupies center stage these days is the three-factor model introduced by Fama and French, briefly discussed in Chapters 11 and 12. The systematic factors in the Fama-French model are firm size and book-to-market ratio as well as the market index.
THE FAMA-FRENCH THREE-FACTOR MODEL

- These additional factors are empirically motivated by the observations that historical-average returns on stocks of small firms and on stocks with high ratios of book equity to market equity (B/M) are higher than predicted by the security market line of the CAPM.
THE FAMA-FRENCH THREE-FACTOR MODEL

- These observations suggest that size or the book-to-market ratio may be proxies for exposures to sources of systematic risk not captured by the CAPM beta and thus result in the return premiums we see associated with these factors.
To create portfolios that track the firm size and book-to-market factors, Davis, Fama, and French (DFF) sorted industrial firms annually by size (market capitalization) and by book-to-market (B/M) ratio. The small-firm group (group S) includes all firms with capitalization below the median, while big (group B) firms have above-median capitalization.
Similarly, firms are annually sorted into three groups based on book-to-market ratio: a low-ratio group (group L) with the 33% lowest B/M ratio, a medium-ratio group (group M), and a high-ratio group (group H). The high-ratio firms often are called *value firms*, since they appear to be “good values,” selling at low multiples of book value.
The intersections of the two size groups with the three value groups results in six groups of firms. Six such portfolios (S/L, S/M, S/H, B/L, B/M, B/H) were constructed each year throughout the period, and the monthly returns of each were recorded. This procedure generated six time series of monthly returns for the years 1929 to 1997.
THE FAMA-FRENCH THREE-FACTOR MODEL

- For each year, the size premium is constructed as the difference in returns between small and large firms. Specifically, the difference in returns of an equally-weighted position in the three small-firm portfolios and an equally-weighted short position in the three big-firm portfolios are computed. SMB (for small minus big) is the return difference.
THE FAMA-FRENCH THREE-FACTOR MODEL

Thus, SMB is calculated from the monthly returns of the six portfolios as

\[
SMB = \frac{1}{3}(S/L + S/M + S/H) - \frac{1}{3}(B/L + B/M + B/H)
\]

(13.8a)
Similarly, the book-to-market effect is calculated from the difference in returns between high B/M ratio and low B/M ratio firms. HML (for high minus low ratio) is constructed as the difference in returns between an equally weighted long position in the high B/M portfolios and an equally weighted short position in the low B/M portfolios.
THE FAMA-FRENCH THREE-FACTOR MODEL

- The monthly values of HML were calculated from the monthly returns on the low and high B/M portfolio as

\[ HML = \frac{1}{2}(S/H + B/H) - \frac{1}{2}(S/L + B/L) \]

(13.8b)
The monthly returns on the market portfolio were calculated from the value-weighted portfolio of all firms listed on the NYSE, AMEX, and Nasdaq. The risk-free rate was the return on 1-month T-bills.
The Fama-French three-factor asset pricing model equation is

\[ E(r_i) - r_f = a_i + b_i[E(r_M) - r_f] + s_iE(SMB) + h_iE(HML) \]  

(13.9a)

The coefficients \( b_i, s_i \) and \( h_i \) are the factor loadings (equivalently, factor betas) on the three relevant risk factors.
According to the APT, the intercept $a_i$ should be zero, since a portfolio with zero loading on all three factors should have an expected excess return of zero.
THE FAMA-FRENCH THREE-FACTOR MODEL

- This equation is estimated as a first-pass regression for each portfolio over the 816 months between 1929 and 1997 using the regression model

\[ r_i - r_f = a_i + b_i (r_M - r_f) + s_i \text{SMB} + h_i \text{HML} + e_i \]

(13.9b)
The Fama-French Three-Factor Model

- Summary statistics for the market excess return, the factor portfolio returns, and the six size-value portfolios are shown in Table 13.4.
<table>
<thead>
<tr>
<th></th>
<th>$R_m - R_f$</th>
<th>SMB</th>
<th>HML</th>
<th>S/L</th>
<th>S/M</th>
<th>S/H</th>
<th>B/L</th>
<th>B/M</th>
<th>B/H</th>
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<tr>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ave</td>
<td>0.67</td>
<td>0.20</td>
<td>0.46</td>
<td>1.05</td>
<td>1.30</td>
<td>1.53</td>
<td>0.89</td>
<td>1.04</td>
<td>1.34</td>
</tr>
<tr>
<td>Std</td>
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<td>3.26</td>
<td>3.11</td>
<td>7.89</td>
<td>7.49</td>
<td>8.38</td>
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<td>6.19</td>
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</tr>
<tr>
<td>$(t)$ (Ave)</td>
<td>3.34</td>
<td>1.78</td>
<td>4.24</td>
<td>3.80</td>
<td>4.96</td>
<td>5.21</td>
<td>4.52</td>
<td>4.78</td>
<td>5.16</td>
</tr>
<tr>
<td>7/29–6/63: 408 months</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<td></td>
</tr>
<tr>
<td>Ave</td>
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<td>0.19</td>
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<td>1.22</td>
<td>1.49</td>
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<td>1.01</td>
<td>1.40</td>
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<td>3.59</td>
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<td>10.57</td>
<td>6.50</td>
<td>7.73</td>
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<td>2.80</td>
<td>2.44</td>
<td>2.71</td>
<td>2.85</td>
<td>2.52</td>
<td>2.64</td>
<td>2.98</td>
</tr>
<tr>
<td>7/63–6/97: 408 months</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ave</td>
<td>0.52</td>
<td>0.21</td>
<td>0.43</td>
<td>1.01</td>
<td>1.38</td>
<td>1.57</td>
<td>0.98</td>
<td>1.06</td>
<td>1.27</td>
</tr>
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<td>Std</td>
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<td>6.60</td>
<td>5.38</td>
<td>4.65</td>
<td>4.12</td>
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<tr>
<td>$(t)$ (Ave)</td>
<td>2.44</td>
<td>1.53</td>
<td>3.38</td>
<td>3.10</td>
<td>5.17</td>
<td>5.88</td>
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<td>5.20</td>
<td>5.87</td>
</tr>
<tr>
<td>7/73–12/93: 246 months</td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
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<td>0.50</td>
<td>1.23</td>
<td>1.60</td>
<td>1.76</td>
<td>0.96</td>
<td>1.20</td>
<td>1.44</td>
</tr>
<tr>
<td>Std</td>
<td>4.79</td>
<td>2.75</td>
<td>2.74</td>
<td>6.88</td>
<td>5.64</td>
<td>5.68</td>
<td>5.22</td>
<td>4.53</td>
<td>4.67</td>
</tr>
<tr>
<td>$(t)$ (Ave)</td>
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<td>1.88</td>
<td>2.87</td>
<td>2.81</td>
<td>4.46</td>
<td>4.87</td>
<td>2.90</td>
<td>4.17</td>
<td>4.83</td>
</tr>
</tbody>
</table>

The data clearly indicate that the small firms and high book-to-market firms have significantly higher average returns. SMB and HML are both reliably positive. Portfolios of small firm (S) and value firms (H) earned significantly higher average returns.
To estimate the regressions (13.9b) for a somewhat larger number of portfolios (nine) and secure an equal number of stocks in each group to equalize diversification, DFF repeat the sorts on both size and B/M, but this time they allocate a third of the sample to small, medium, and large firms in the size sort, as well as to low, medium, and high B/M ratio firms in an independent sort.
THE FAMA-FRENCH THREE-FACTOR MODEL

- The intersections of these groups lead to nine portfolios containing equal numbers of stocks.
- The estimation results of the nine regressions (equation 13.9b) for the overall sample period and for the early and late half-periods are presented in Table 13.5.
<table>
<thead>
<tr>
<th></th>
<th>BE/ME</th>
<th>Size</th>
<th>Ex Ret</th>
<th>a</th>
<th>b</th>
<th>s</th>
<th>h</th>
<th>t(a)</th>
<th>t(b)</th>
<th>t(s)</th>
<th>t(h)</th>
<th>R^2</th>
</tr>
</thead>
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<td>7/29-6/97</td>
<td></td>
<td></td>
<td></td>
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<td></td>
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<td></td>
<td></td>
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<td></td>
</tr>
<tr>
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<td>0.91</td>
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<td>1.16</td>
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<td>19.49</td>
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<td>0.96</td>
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<td>1.03</td>
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<td>0.77</td>
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<td>39.21</td>
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<td>0.98</td>
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<td>M/L</td>
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<td>0.70</td>
<td>-0.06</td>
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<tr>
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<td>0.97</td>
</tr>
<tr>
<td>B/L</td>
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<td>94.65</td>
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<td>0.14</td>
<td>0.34</td>
<td>1.76</td>
<td>61.61</td>
<td>13.66</td>
<td>21.02</td>
<td>0.93</td>
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<td>7/29-6/63</td>
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<tr>
<td>S/L</td>
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<td>23.83</td>
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<td>-0.53</td>
<td>1.01</td>
<td>1.47</td>
<td>0.23</td>
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<td>1.24</td>
<td>0.38</td>
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<td>34.72</td>
<td>15.60</td>
<td>6.21</td>
<td>0.95</td>
</tr>
<tr>
<td>S/H</td>
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<td>1.02</td>
<td>1.17</td>
<td>0.83</td>
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<td>44.71</td>
<td>28.80</td>
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<td>0.98</td>
</tr>
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<td>M/L</td>
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<td>0.98</td>
<td>0.56</td>
<td>0.01</td>
<td>-1.14</td>
<td>37.44</td>
<td>12.26</td>
<td>0.39</td>
<td>0.96</td>
</tr>
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<td>M/M</td>
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<td>1.13</td>
<td>0.00</td>
<td>1.07</td>
<td>0.47</td>
<td>0.33</td>
<td>0.07</td>
<td>26.38</td>
<td>11.77</td>
<td>7.73</td>
<td>0.97</td>
</tr>
<tr>
<td>M/H</td>
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<td>51.59</td>
<td>1.30</td>
<td>-0.07</td>
<td>1.07</td>
<td>0.50</td>
<td>0.79</td>
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<td>94.92</td>
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<td>1.02</td>
<td>0.06</td>
<td>0.20</td>
<td>0.20</td>
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<tr>
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<td>0.12</td>
<td>0.37</td>
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<td>2.90</td>
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<td>1.30</td>
<td>0.00</td>
<td>1.02</td>
<td>0.12</td>
<td>0.97</td>
<td>-0.01</td>
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<td>-0.96</td>
<td>17.99</td>
<td>0.94</td>
</tr>
<tr>
<td>7/63-6/97</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>S/L</td>
<td>0.42</td>
<td>20.94</td>
<td>0.54</td>
<td>-0.22</td>
<td>1.06</td>
<td>1.22</td>
<td>0.14</td>
<td>-3.31</td>
<td>60.47</td>
<td>39.87</td>
<td>-4.51</td>
<td>0.96</td>
</tr>
<tr>
<td>S/M</td>
<td>0.87</td>
<td>20.68</td>
<td>0.89</td>
<td>0.03</td>
<td>0.97</td>
<td>1.02</td>
<td>0.31</td>
<td>0.71</td>
<td>74.53</td>
<td>52.41</td>
<td>13.82</td>
<td>0.98</td>
</tr>
<tr>
<td>S/H</td>
<td>1.71</td>
<td>17.88</td>
<td>1.04</td>
<td>0.04</td>
<td>0.99</td>
<td>1.03</td>
<td>0.62</td>
<td>1.27</td>
<td>75.12</td>
<td>64.49</td>
<td>25.86</td>
<td>0.98</td>
</tr>
<tr>
<td>M/L</td>
<td>0.42</td>
<td>56.51</td>
<td>0.56</td>
<td>-0.02</td>
<td>1.07</td>
<td>0.58</td>
<td>-0.24</td>
<td>-0.33</td>
<td>71.73</td>
<td>27.08</td>
<td>-9.73</td>
<td>0.96</td>
</tr>
<tr>
<td>M/M</td>
<td>0.87</td>
<td>55.93</td>
<td>0.77</td>
<td>0.02</td>
<td>1.00</td>
<td>0.48</td>
<td>0.30</td>
<td>0.31</td>
<td>64.36</td>
<td>22.60</td>
<td>11.22</td>
<td>0.95</td>
</tr>
<tr>
<td>M/H</td>
<td>1.54</td>
<td>54.83</td>
<td>0.96</td>
<td>0.03</td>
<td>1.05</td>
<td>0.55</td>
<td>0.63</td>
<td>0.53</td>
<td>69.16</td>
<td>28.08</td>
<td>24.23</td>
<td>0.96</td>
</tr>
<tr>
<td>B/L</td>
<td>0.38</td>
<td>94.38</td>
<td>0.45</td>
<td>0.10</td>
<td>0.99</td>
<td>-0.15</td>
<td>-0.32</td>
<td>2.89</td>
<td>91.73</td>
<td>-8.92</td>
<td>-16.53</td>
<td>0.98</td>
</tr>
<tr>
<td>B/M</td>
<td>0.86</td>
<td>92.14</td>
<td>0.54</td>
<td>-0.04</td>
<td>0.99</td>
<td>-0.19</td>
<td>0.25</td>
<td>0.70</td>
<td>55.19</td>
<td>-6.91</td>
<td>8.53</td>
<td>0.91</td>
</tr>
<tr>
<td>B/H</td>
<td>1.41</td>
<td>90.16</td>
<td>0.70</td>
<td>-0.13</td>
<td>1.04</td>
<td>-0.01</td>
<td>0.69</td>
<td>-2.59</td>
<td>78.64</td>
<td>-0.36</td>
<td>28.53</td>
<td>0.94</td>
</tr>
</tbody>
</table>

THE FAMA-FRENCH THREE-FACTOR MODEL

- The $R^2$-square statistics, all in excess of 0.91, show that returns are well explained by the three factor portfolios, and the $t$-statistics of the loadings on the size and value factors show that these factors contribute significantly to explanatory power. Moreover, for the full sample period, only one of the nine portfolios has an economically and statistically significant intercept.
How should we interpret these results?

One argument is that size and relative value (as measured by the B/M ratio) proxy for risk not captured by the CAPM beta alone. This explanation is consistent with the APT in that it implies that size and value are priced risk factors and, hence, that these premiums do not represent mispricing.
Another explanation attributes these premiums to irrational investor preferences for large size or low B/M firms, which drives up their prices and drives down the returns on these firms. This question may be more relevant for the B/M or “value” factor in light of evidence that the size premium has largely vanished in recent years.
The question about the source of the value premium must be resolved by testing whether, within each group, portfolios with larger loadings on the value factor (larger $h$ coefficient on the HML portfolio) do indeed earn a higher average return.
THE FAMA-FRENCH THREE-FACTOR MODEL

- If they do not, then it appears that firms with these characteristics (i.e., a higher ratio of B/M) earn higher returns but that a greater exposure to the B/M factor per se does not itself predict higher returns. This would be evidence of mispricing.
To address this issue, DFF sort each of the portfolios into three subgroups according to their $h$ value over the previous 3 years and show that higher $h$ portfolios (with greater exposure to HML) earned a higher average return over the full sample period. But DFF also found that this was not the case in the 20-year period between 1973 and 1993.
Thus, the important question of whether the model is consistent with a three-factor version of the APT, or instead suggests irrational investor behavior, is not categorically resolved. See the nearby box for further recent approaches to the risk-return tradeoff.
TIME-VARYING VOLATILITY

- We may associate the variance of the rate of return on the stock with the rate of arrival of new information because new information may lead investors to revise their assessment of intrinsic value.
As a casual survey of the media would indicate, the rate of revision in predictions of business cycles, industry ascents or descents, and the fortunes of individual enterprises fluctuates regularly; in other words, the rate of arrival of new information is time varying.
TIME-VARYING VOLATILITY

- Consequently, we should expect the variances of the rates of return on stocks (as well as the covariances among them) to be time varying.
TIME-VARYING VOLATILITY

- In an exploratory study of the volatility of NYSE stocks over more than 150 years (using monthly returns over 1835-1987), Pagan and Schwert computed estimates of the variance of monthly returns. Their results, depicted in Figure 13.3, show just how important it may be to consider time variation in stock variance.
Figure 13.3 Estimates of the monthly stock return variance, 1835–1987

The centrality of the risk-return trade-off suggests that once we make sufficient progress in the modeling, estimation, and prediction of the time variation in return variances and covariances, we should expect a significant refinement in our understanding of expected returns as well.
When we consider a time-varying return distribution, we must refer to the conditional mean, variance, and covariance, that is, the mean, variance, or covariance conditional on currently available information. The “conditions” that vary over time are the values of variables that determine the level of these parameters.
TIME-VARYING VOLATILITY

- In contrast, the usual estimate of return variance, the average of squared deviations over the sample period, provides an unconditional estimate, because it treats the variance as constant over time.
In 1982 Robert F. Engle published a study of U.K. inflation rates that measured their time-varying volatility. His model, named ARCH (autoregressive conditional heteroskedasticity), is based on the idea that a natural way to update a variance forecast is to average it with the most recent squared “surprise” (i.e., the squared deviation of the rate of return from its mean).
TIME-VARYING VOLATILITY

Today, the most widely used model to estimate the conditional (hence time-varying) variance of stocks and stock-index returns is the generalized autoregressive conditional heteroskedasticity (CARCH) model, also pioneered by Robert F. Engle. (The generalized ARCH model allows greater flexibility in the specification of how volatility evolves over time.)
TIME-VARYING VOLATILITY

- The GARCH model uses rate-of-return history as the information set used to form our estimates of variance. The model posits that the forecast of market volatility evolves relatively smoothly each period in response to new observations on market returns.
The updated estimate of market-return variance in each period depends on both the previous estimate and the most recent squared residual return on the market.
TIME-VARYING VOLATILITY

- The squared residual is an unbiased estimate of variance, so this technique essentially mixes in a statistically efficient manner the previous volatility estimate with an unbiased estimate based on the new observation of market return.
The updating formula is

\[ \sigma_t^2 = a_0 + a_1 \varepsilon_{t-1}^2 + a_2 \sigma_{t-1}^2 \]  

(13.10)
As noted, equation 13.8 asserts that the updated forecast of variance is a function of the most recent variance forecast $\sigma_{t-1}^2$, and the most recent squared prediction error in market return, $\varepsilon_{t-1}^2$. The parameters $a_0$, $a_1$, and $a_2$ are estimated from past data.
Evidence on the relationship between mean and variance has been mixed. Whitelaw found that average returns and volatility are negatively related, but Kane, Marcus, and Noh found a positive relationship.
ARCH-type models clearly capture much of the variation in stock market volatility. Figure 13.4 compares volatility estimates form an ARCH model to volatility estimates derived from prices on market-index options. The variation in volatility, as well as the close agreement between the estimates, is evident.
In an intriguing article Mehra and Prescott examined the excess returns earned on equity portfolios over the risk-free rate during the period 1889–1978. They concluded that the historical-average excess return has been too large to be consistent with reasonable levels of risk aversion.
In other words, it appears that the reward investors have received for bearing risk has been so generous that it is hard to reconcile with rational security pricing. This research has since engendered a large body of literature attempting to explain this puzzle.
Two recent explanations for the puzzle deserve special attention, as they utilize important insights into the difficulties of obtaining inferences from observations of realized returns.
Fama and French offer one possible interpretation of the puzzle. They use the sample period, 1872-1999, and report the average risk-free rates, average return on equity (represented by the S&P 500 index), and the resultant risk premium for the overall period and subperiods:
### Expected versus Realized Returns

<table>
<thead>
<tr>
<th>Period</th>
<th>Risk-Free Rate</th>
<th>S&amp;P 500</th>
<th>Equity Premium</th>
</tr>
</thead>
<tbody>
<tr>
<td>1872–1999</td>
<td>4.87</td>
<td>10.97</td>
<td>6.10</td>
</tr>
<tr>
<td>1872–1949</td>
<td>4.05</td>
<td>8.67</td>
<td>4.62</td>
</tr>
<tr>
<td>1950–1999</td>
<td>6.15</td>
<td>14.56</td>
<td>8.41</td>
</tr>
</tbody>
</table>
Expected versus Realized Returns

- The difference in results before and after 1949 suggests that the equity premium puzzle is really a creature of modern times.
Expected versus Realized Returns

- Fama and French (FF) suspect that estimating the risk premium from average realized returns may be the problem. They use the constant-growth dividend-discount model to estimate expected returns and find that for the period 1872–1949, the dividend discount model (DDM) yields similar estimates of the expected risk premium as the average realized excess return.
Expected versus Realized Returns

But for the period 1950–1999, the DDM yields a much smaller risk premium, which suggests that the high average excess return in this period may have exceeded the returns investors actually expected to earn at the time.
In the constant-growth DDM, the expected capital gains rate on the stock will equal the growth rate of dividends. As a result, the expected total return on the firm’s stock will be the sum of dividend yield (dividend/price) plus the expected dividend growth rate, \( g \): (next slide)
Expected versus Realized Returns

\[ E(r) = \frac{D_1}{P_0} + g \]  \hspace{1cm} (13.11)

where \( D_1 \) is end-of-year dividends and \( P_0 \) is the current price of the stock. Fama and French treat the S&P 500 as representative of the average firm, and use equation 13.11 to produce estimates of \( E(r) \).
Expected versus Realized Returns

For any sample period, $t = 1,\ldots, T$. Fama and French estimate expected return from the arithmetic average of the dividend yield ($D_t/P_{t-1}$) plus the dividend growth rate ($g_t = D_t/D_{t-1}$). In contrast, the realized return is the dividend yield plus the rate of capital gains ($P_t/P_{t-1} - 1$).
Because the dividend yield is common to both estimates, the difference between the expected and realized return equals the difference between the dividend growth and capital gains rates. While dividend growth and capital gains were similar in the earlier period, capital gains significantly exceeded the dividend growth rate in modern times.
Expected versus Realized Returns

- Hence, FF conclude that the equity premium puzzle may be due at least in part to unanticipated capital gains in the latter period.
FF argue that dividend growth rates produce more reliable estimates of expected capital gains than the average of realized capital gains. They point to three reasons: (next three slides)
1. Average realized returns over 1950–1999 exceeded the internal rate of return on corporate investments. If those returns were representative of expectations, we would have to conclude that firms were willingly engaging in negative NPV investments.
2. The statistical precision of estimates from the DDM are far higher than those using average historical returns. The standard error of the estimates of the risk premium from realized returns is about 2.5 times the standard error form the dividend discount model (see the following table).
3. The reward-to-variability (Sharpe) ratio derived from the DDM is far more stable than that derived from realized returns. If risk aversion remains the same over time, we would expect the Sharpe ratio to be stable.
The evidence for the second and third points is shown in the following table, where estimates from the dividend model (DDM) and from realized returns (Realized) are shown side by side.
## Expected versus Realized Returns

<table>
<thead>
<tr>
<th>Period</th>
<th>Mean Return</th>
<th>Standard Error</th>
<th>t-statistic</th>
<th>Sharpe Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>DDM</td>
<td>Realized</td>
<td>DDM</td>
<td>Realized</td>
</tr>
<tr>
<td>1872-1999</td>
<td>4.03</td>
<td>6.10</td>
<td>1.14</td>
<td>1.65</td>
</tr>
<tr>
<td>1872-1949</td>
<td>4.35</td>
<td>4.62</td>
<td>1.76</td>
<td>2.20</td>
</tr>
<tr>
<td>1950-1999</td>
<td>3.54</td>
<td>8.41</td>
<td>1.03</td>
<td>2.45</td>
</tr>
</tbody>
</table>

### Notes
- **Mean Return**: Average return over the specified period.
- **Standard Error**: Measure of variability around the mean.
- **t-statistic**: Statistic used to determine the significance of the difference between two means.
- **Sharpe Ratio**: Ratio of return to risk.
Expected versus Realized Returns

- FF’s innovative study thus provides a possible explanation of the equity premium puzzle. Another implication from the study may be even more important for today’s investor: The study predicts that future excess returns will be significantly lower than those experienced in recent decades.
The equity premium puzzle emerged from long-term averages of U.S. stock returns. There are reasons to suspect that these estimates of the risk premium are subject to survivorship bias, as the United States has arguably been the most successful capitalist system in the world, an outcome that probably would not have been anticipated several decades ago.
Jurion and Goetzmann assembled a database of capital appreciation indexes for the stock markets of 39 countries over the period 1926–1996.

Figure 13.5 shows that U.S. equities had the highest real return of all countries, at 4.3% annually, versus a median of 0.8% for other countries.
Figure 13.5
Real returns on global stock markets. The figure displays average real returns for 39 markets over the period 1921 to 1996. Markets are sorted by years of existence. The graph shows that markets with long histories typically have higher returns. An asterisk indicates that the market suffered a long-term break.
Moreover, unlike the United States, many other countries have had equity markets that actually closed, either permanently, or for extended periods of time.
Survivorship Bias

- The implication of these results is that using average U.S. data may induce a form of survivorship bias to our estimate of expected returns, since unlike many other countries, the United States has never been a victim of such extreme problems.
Survivorship Bias

- Estimating risk premiums from the experience of the most successful country and ignoring the evidence from stock markets that did not survive for the full sample period will impart an upward bias in estimates of expected returns. The high realized equity premium obtained for the United States may not be indicative of required returns.
Survivorship Bias

- As an analogy, think of the effect of survivorship bias in the mutual fund industry. We know that some companies regularly close down their worst-performing mutual funds.
Survivorship Bias

- If performance studies include only mutual funds for which returns are available during an entire sample period, the average measured performance of mutual fund managers will be better than one could reasonably expect from the full sample of managers.
Survivorship Bias

Think back to the box in Chapter 12, “How to Guarantee a Successful Market Newsletter.” If one starts many newsletters with a range of forecasts, and continues only the newsletters that turned out to have successful advice, then it will *appear* from the sample of survivors that the average newsletter had forecasting skill.
SURVIVORSHIP BIAS AND TESTS OF MARKET EFFICIENCY

- We’ve seen that survivorship bias might be one source of the equity premium puzzle. It turns out that survivorship bias also can affect our measurement of persistence in stock market returns, an issue that is crucial for tests of market efficiency.
SURVIVORSHIP BIAS AND TESTS OF MARKET EFFICIENCY

- For a demonstration of the potential impact of survivorship bias, imagine that a new group of mutual funds is set up. Half the funds are managed aggressively and the other conservatively; however, none of the managers are able to beat the market in expectation.
Survivorship bias and tests of market efficiency

- The probability distribution of alpha values is given by

<table>
<thead>
<tr>
<th>Alpha Value (%)</th>
<th>Probability</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Conservative Manager</td>
<td>Aggressive Manager</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>0.5</td>
</tr>
<tr>
<td>1</td>
<td>0.5</td>
<td>0</td>
</tr>
<tr>
<td>-1</td>
<td>0.5</td>
<td>0</td>
</tr>
<tr>
<td>-3</td>
<td>0</td>
<td>0.5</td>
</tr>
</tbody>
</table>
Because there are an equal number of aggressive and conservative managers, the frequency distribution of alphas in a given period is: (next slide)
<table>
<thead>
<tr>
<th>Alpha Value (%)</th>
<th>Relative Frequency of Funds</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Conservative Manager</td>
<td>Aggressive Manager</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>.25</td>
</tr>
<tr>
<td>1</td>
<td>.25</td>
<td>0</td>
</tr>
<tr>
<td>-1</td>
<td>.25</td>
<td>0</td>
</tr>
<tr>
<td>-3</td>
<td>0</td>
<td>.25</td>
</tr>
<tr>
<td>Total</td>
<td>.5</td>
<td>.5</td>
</tr>
</tbody>
</table>
Define a “winner” fund as one in the top-half of the distribution of returns in a given period; a “loser” is one in the bottom half of the sample. Manager alphas are assumed to be serially uncorrelated. Therefore, the probability of being a winner or a loser in the second quarter is the same regardless of first-quarter performance.
SURVIVORSHIP BIAS AND TESTS OF MARKET EFFICIENCY

- A 2 × 2 tabulation of performance in two consecutive periods, such as in the following table, will show absence of any persistence in performance.

<table>
<thead>
<tr>
<th></th>
<th>Second Period</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Winners</td>
</tr>
<tr>
<td>First Period</td>
<td></td>
</tr>
<tr>
<td>Winners</td>
<td>.25</td>
</tr>
<tr>
<td>Losers</td>
<td>.25</td>
</tr>
</tbody>
</table>
SURVIVORSHIP BIAS AND TESTS OF MARKET EFFICIENCY

- But now assume that in each quarter funds are ranked by returns and the bottom 5% are closed down. A researcher obtains a sample of four quarters of fund returns and ranks the semiannual performance of funds that survived the entire sample.
SURVIVORSHIP BIAS AND TESTS OF MARKET EFFICIENCY

The following table, based only on surviving funds, seems to show that first-period winners are far more likely to be second-period winners as well.

<table>
<thead>
<tr>
<th>First Period</th>
<th>Second Period</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Winners</td>
<td>Losers</td>
<td>Row Total</td>
</tr>
<tr>
<td>Winners</td>
<td>.3893</td>
<td>.1401</td>
<td>.5294</td>
</tr>
<tr>
<td>Losers</td>
<td>.1107</td>
<td>.4706</td>
<td>.4706</td>
</tr>
<tr>
<td>Column total</td>
<td>.5001</td>
<td>.4999</td>
<td>1.0000</td>
</tr>
</tbody>
</table>
The degree of survivorship bias depends first and foremost on how aggressively poorly performing funds are shut down. (In this example, the worst 5% of performers were shut down.) The bias increases enormously with the cut-off rate.
SURVIVORSHIP BIAS AND TESTS OF MARKET EFFICIENCY

- Other factors affecting bias are correlation across manager portfolios, serial correlation of returns, the dispersion of style across managers, and the strategic response of managers to the possibility of cut-off.
To assess the potential effect of actual survivorship bias, Brown, Goetzmann, Ibbotson, and Ross conducted a simulation using observed characteristics of mutual fund returns. Their results demonstrate that actual survivorship bias could be strong enough to create apparent persistence in the performance of portfolio managers.
They simulate annual returns over a 4-year period for 600 mutual fund managers, drawing from distributions that are constructed to mimic observed equity returns in the United States over the period 1926–1989, and mutual fund returns reported in a performance study by Goetzmann and Ibbotson.
SURVIVORSHIP BIAS AND TESTS OF MARKET EFFICIENCY

- Four annual returns for each manager are generated independently so that relative performance over the first 2-year period does not persist in the following 2-year period. The simulated returns of the funds and the market index are used to compute four risk-adjusted annual returns (alphas) for each of the 600 managers. Winners (losers) are identified by positive (negative) alphas.
Two-by-two tabulations of the frequency of first-period/second-period winners and losers are shown in Table 13.6.
### Table 13.6 Two-Way Table of Managers Classified by Risk-Adjusted Returns over Successive Intervals, a Summary of 20,000 Simulations Assuming 0, 5, 10, and 20% Cut-offs

<table>
<thead>
<tr>
<th></th>
<th>Second-Period Winners</th>
<th>Second-Period Losers</th>
</tr>
</thead>
<tbody>
<tr>
<td>No cut-off (n = 600)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>First-period winners</td>
<td>150.09</td>
<td>149.51</td>
</tr>
<tr>
<td>First-period losers</td>
<td>149.51</td>
<td>150.09</td>
</tr>
<tr>
<td>Average cross-section (t)-value = -0.004</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Average annual excess return = 0.0%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Average (\beta) = 0.950</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5% cut-off (n = 494)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>First-period winners</td>
<td>127.49</td>
<td>119.51</td>
</tr>
<tr>
<td>First-period losers</td>
<td>119.51</td>
<td>127.49</td>
</tr>
<tr>
<td>Average cross-section (t)-value = 2.046</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Average annual excess return = 0.44%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Average (\beta) = 0.977</td>
<td></td>
<td></td>
</tr>
<tr>
<td>10% cut-off (n = 398)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>First-period winners</td>
<td>106.58</td>
<td>92.42</td>
</tr>
<tr>
<td>First-period losers</td>
<td>92.42</td>
<td>106.58</td>
</tr>
<tr>
<td>Average cross-section (t)-value = 3.356</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Average annual excess return = 0.61%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Average (\beta) = 0.994</td>
<td></td>
<td></td>
</tr>
<tr>
<td>20% cut-off (n = 249)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>First-period winners</td>
<td>71.69</td>
<td>53.31</td>
</tr>
<tr>
<td>First-period losers</td>
<td>53.31</td>
<td>70.69</td>
</tr>
<tr>
<td>Average cross-section (t)-value = 4.679</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Average annual excess return = 0.80%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Average (\beta) = 1.018</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

In each of the 4 years, managers who experience returns in the lowest percentile indicated by the cut-off value are excluded from the sample, and this experiment is repeated 20,000 times. Thus, the numbers in the first \(2 \times 2\) table give the average frequency with which the 600 managers fall into the respective classifications. The second panel shows the average frequencies for the 494 managers who survive the performance cut, while the third and fourth panels give corresponding results for 398 and 249 managers. For each simulation, the winners are defined as those managers whose average 2-year Jensen’s \(\alpha\) measure was greater than or equal to that of the median manager in that sample.
When all 600 managers are included in the 4-year sample, no persistence in performance can be detected. But when the poor performers in each year are eliminated from the sample, performance persistence shows up. Elimination of even a small number of poor performers can generate a significant level of apparent persistence.
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- The result for a 5% cut-off rate in Table 13.6 are not as strong as in the “clean” example we presented above; apparently, other factors mitigate the effect somewhat. Still, survivorship bias is sufficient to create an appearance of significant performance persistence even when actual returns are consistent with efficient markets.