CHAPTER 5

HISTORY OF INTEREST RATES AND RISK PREMIUMS
HISTORY OF INTEREST RATES AND RISK PREMIUMS

- Determinants of the Level of Interest Rates
- Risk and Risk Premium
- The Historical Record
- Real Versus Nominal Risk
HISTORY OF INTEREST RATES AND RISK PREMIUMS

- Return Distributions and Value at Risk
- A Global View of the Historical Record
- Forecasts for the Long Haul
DETERMINANTS OF THE LEVEL OF INTEREST RATES

- Interest rates and forecasts of their future values are among the most important inputs into an investment decision.
For example, suppose you have $10,000 in a savings account. The bank pays you a variable interest rate tied to some short-term reference rate such as the 30-day Treasury bill rate. You have the option of moving some or all of your money into a longer-term certificate of deposit that offers a fixed rate over the term of the deposit.
Your decision depends critically on your outlook for interest rates.

- If you think rates will fall, you will want to lock in the current higher rates by investing in a relatively long-term CD.
- If you expect rates to rise, you will want to postpone committing any funds to long-term CDs.
Forecasting interest rates is one of the most notoriously difficult parts of applied macroeconomics. Nonetheless, we do have a good understanding of the fundamental factors that determine the level of interest rates:
DETERMINANTS OF THE LEVEL OF INTEREST RATES

1. The supply of funds from savers, primarily households.
2. The demand for funds from businesses to be used to finance investments in plant, equipment, and inventories (real assets or capital formation).
3. The government’s net supply of or demand for funds as modified by actions of the Federal Reserve Bank.
Real and Nominal Rates of Interest

Suppose exactly 1 year ago you deposited $1,000 in a 1-year time deposit guaranteeing a rate of interest of 10%. You are about to collect $1,100 in cash.
Real and Nominal Rates of Interest

- Is your $100 return for real?
  - That depends on what money can buy these days, relative to what you *could* buy a year ago. The consumer price index (CPI) measures purchasing power by averaging the prices of goods and services in the consumption basket of an average urban family of four. Although this basket may not represent your particular consumption plan, suppose for now that it does.
Real and Nominal Rates of Interest

Suppose the rate of inflation (the percent change in the CPI, denoted by $i$) for the last year amounted to $i = 6\%$. This tells you that the purchasing power of money is reduced by 6\% a year. The value of each dollar depreciates by 6\% a year in terms of the goods it can buy.
Real and Nominal Rates of Interest

- Therefore, part of your interest earnings are offset by the reduction in the purchasing power of the dollars you will receive at the end of the year.

- With a 10% interest rate, after you net out the 6% reduction in the purchasing power of money, you are left with a net increase in purchasing power of about 4%.
Real and Nominal Rates of Interest

Thus we need to distinguish between a nominal interest rate—the growth rate of your money—and a real interest rate—the growth rate of your purchasing power.
Real and Nominal Rates of Interest

- If we call $R$ the nominal rate, $r$ the real rate, and $i$ the inflation rate, then we conclude

$$r \approx R - i$$

In words, the real rate of interest is the nominal rate reduced by the loss of purchasing power resulting from inflation.
Real and Nominal Rates of Interest

- In fact, the exact relationship between the real and nominal interest rate is given by

\[ 1 + r = \frac{1 + R}{1 + i} \]

This is because the growth factor of your purchasing power, \(1 + r\), equals the growth factor of your money, \(1 + R\), divided by the new price level, that is, \(1 + i\) times its value in the previous period.
The exact relationship can be rearranged to

\[ r = \frac{(R - i)}{(1 + i)} \]

which shows that the approximation rule overstates the real rate by the factor \(1 + i\).
EXAMPLE 5.1: Approximating the Real Rate

- If the nominal interest rate on a 1-year CD is 8%, and you expect inflation to be 5% over the coming year, then
  - using the approximation formula, you expect the real rate of interest to be \( r = 8\% - 5\% = 3\% \).
  - using the exact formula, the real rate is \( r = (0.08 - 0.05)/(1 + 0.05) = 0.0286 \), or 2.86%.
  - Therefore, the approximation rule overstates the expected real rate by only, .14% (14 basis points).
Real and Nominal Rates of Interest

- Before the decision to invest, you should realize that conventional certificates of deposit offer a guaranteed *nominal* rate of interest. Thus you can only infer the expected real rate on these investments by subtracting your expectation of the rate of inflation.
Real and Nominal Rates of Interest

- It is always possible to calculate the real rate after the fact. The inflation rate is published by the Bureau of Labor Statistics (BLS). The future real rate, however, is unknown, and one has to rely on expectations. In other words, because future inflation is risky, the real rate of return is risky even when the nominal rate is risk-free.
The Equilibrium Real Rate of Interest

- Three basic factors—supply, demand, and government actions—determine the real interest rate.
- The nominal interest rate, which is the rate we actually observe, is the real rate plus the expected rate of inflation.
- So a fourth factor affecting the interest rate is the expected rate of inflation.
Although there are many different interest rates economywide, these rates tend to move together, so economists frequently talk as if there were a single representative rate. We can use this abstraction to gain some insights into the real rate of interest if we consider the supply and demand curves for funds.
The Equilibrium Real Rate of Interest

- Figure 5.1 shows a downward-sloping demand curve and an upward-sloping supply curve.
  - On the horizontal axis, we measure the quantity of funds.
  - On the vertical axis, we measure the real rate of interest.
Figure 5.1
Determination of the equilibrium real rate of interest
The Equilibrium Real Rate of Interest

• The supply curve slopes up from left to right because the higher the real interest rate, the greater the supply of household savings.
  - The assumption is that at higher real interest rates households will choose to postpone some current consumption and set aside or invest more of their disposable income for future use.
The Equilibrium Real Rate of Interest

- The demand curve slopes down from left to right because the lower the real interest rate, the more businesses will want to invest in physical capital.
  - Assuming that businesses rank projects by the expected real return on invested capital, firms will undertake more projects the lower the real interest rate on the funds needed to finance those projects.
The Equilibrium Real Rate of Interest

- Equilibrium is at the point of intersection of the supply and demand curves, point $E$ in Figure 5.1.
The Equilibrium Real Rate of Interest

- The government and the central bank (the Federal Reserve) can shift these supply and demand curves either to the right or to the left through fiscal and monetary policies.
The Equilibrium Real Rate of Interest

- Fox example, consider an increase in the government’s budget deficit. This increases the government’s borrowing demand and shifts the demand curve to the right, which causes the equilibrium real interest rate to rise to point $E'$. That is, a forecast that indicates higher than previously expected government borrowing increases expected future interest rates.
The Equilibrium Real Rate of Interest

- The Fed can offset such a rise through an expansionary monetary policy, which will shift the supply curve to the right.
The Equilibrium Real Rate of Interest

Thus, although the fundamental determinants of the real interest rate are the propensity of households to save and the expected productivity (or we could say profitability) of investment in physical capital, the real rate can be affected as well by government fiscal and monetary policies.
The Equilibrium Nominal Rate of Interest

- We’ve seen that the real rate of return on an asset is approximately equal to the nominal rate minus the inflation rate. Because investors should be concerned with their real returns—the increase in their purchasing power—we would expect that as the inflation rate increases, investors will demand higher nominal rates of return offered by an investment.
The Equilibrium Nominal Rate of Interest

- Irving Fisher (1930) argued that the nominal rate ought to increase one for one with increases in the expected inflation rate.
  - If we use the notation $E(i)$ to denote the current expectation of the inflation rate that will prevail over the coming period, then we can state the so-called *Fisher equation* formally as

  $$ R = r + E(i) $$
The Equilibrium Nominal Rate of Interest

- The Fisher equation implies that if real rates are reasonably stable, then increases in nominal rates ought to predict higher inflation rates.
- This relationship has been debated and empirically investigated. The results are mixed; although the data do not strongly support this relationship, nominal interest rates seem to predict inflation as well as alternative methods, in part because we are unable to forecast inflation well with any method.
The Equilibrium Nominal Rate of Interest

- One reason it is difficult to determine the empirical validity of the Fisher hypothesis that changes in nominal rates predict changes in future inflation rates is that the real rate also changes unpredictably over time. Nominal interest rates can be viewed as the sum of the required real rate on nominally risk-free assets, plus a “noisy” forecast of inflation.
The Equilibrium Nominal Rate of Interest

- Longer rates incorporate forecasts for long-term inflation. For this reason alone, interest rates on bonds of different maturity may diverge.
The Equilibrium Nominal Rate of Interest

- In addition, we will see that prices of longer-term bonds are more volatile than those of short-term bonds. This implies that expected returns on longer-term bonds may include a risk premium, so that the expected real rate offered by bonds of varying maturity also may vary.
The Fisher equation predicts a close connection between inflation and the rate of return on T-bills. This is apparent in Figure 5.2, which plots both time series on the same set of axes.

- Both series tend to move together, which is consistent with our previous statement that expected inflation is a significant force determining the nominal rate of interest.
Figure 5.2  Interest and inflation rates, 1963–2002
For a holding period of 30 days, the difference between actual and expected inflation is not large. The 30-day bill rate will adjust rapidly to changes in expected inflation induced by observed changes in actual inflation. It is not surprising that we see nominal rates on bills move roughly in tandem with inflation over time.
Taxes and the Real Rate of Interest

- Tax liabilities are based on *nominal* income and the tax rate determined by the investor’s tax bracket. Congress recognized the resultant “bracket creep” (when nominal income grows due to inflation and pushes taxpayers into higher brackets) and mandated index-linked tax brackets in the Tax Reform Act of 1986.
Index-linked tax brackets do not provide relief from the effect of inflation on the taxation of savings, however. Given a tax rate \( t \) and a nominal interest rate \( R \), the after-tax interest rate is \( R(1 - t) \). The real after-tax rate is approximately the after-tax nominal rate minus the inflation rate:

\[
R(1 - t) - i = (r + i)(1 - t) - i = r(1 - t) - it
\]
Taxes and the Real Rate of Interest

- Thus the after-tax real rate of return falls as the inflation rate rises. Investors suffer an inflation penalty equal to the tax rate times the inflation rate \((it)\).
If, for example, you are in a 30% tax bracket and your investments yield 12%, while inflation runs at the rate of 8%, then:

- your before-tax real rate is approximately 4%, and you should, in an inflation-protected tax system, net after taxes a real return of $4\%(1 - .3) = 2.8\%$. 
Taxes and the Real Rate of Interest

- But the tax code does not recognize that the first 8% of your return is no more than compensation for inflation—not real income—and hence your after-tax return is reduced by 8% × 0.3 = 2.4%, so that your after-tax real interest rate, at 0.4%, is almost wiped out.
Suppose you are considering investing in a stock market index fund. The price of a share in the fund is currently $100, and your investment horizon is 1 year. You expect the cash dividend during the year to be $4, so your expected dividend yield (dividends earned per dollar invested) is 4%.
Your total holding-period return (HPR) will depend on the price that will prevail 1 year from now. Suppose your best guess is that it will be $110 per share. Then your *capital gain* will be $10 and your HPR will be 14%.
The definition of the holding-period return is capital gain income plus dividend income per dollar invested in the stock:

- \( \text{HPR} = \frac{\text{Ending price of a share} - \text{Beginning price} + \text{Cash dividend}}{\text{Beginning price}} \)

- In our case we have \( \text{HPR} = \frac{\$110 - \$100 + \$4}{\$100} = .14 \) or 14%

- This definition of the HPR assumes the dividend is paid at the end of the holding period.
To the extent that dividends are received earlier, the HPR ignores reinvestment income between the receipt of the payment and the end of the holding period.

Recall also that the percent return from dividends is called the dividend yield, and so the dividend yield plus the capital gains yield equals the HPR.
RISK AND RISK PREMIUMS

- There is considerable uncertainty about the price of a share a year from now, however, so you cannot be sure about your eventual HPR.
- We can try to quantify our beliefs about the state of the economy and the stock market in terms of three possible scenarios with probabilities as presented in Table 5.1.
### Table 5.1 Probability Distribution of HPR on the Stock Market

<table>
<thead>
<tr>
<th>State of the Economy</th>
<th>Probability</th>
<th>Ending Price</th>
<th>HPR</th>
</tr>
</thead>
<tbody>
<tr>
<td>Boom</td>
<td>.25</td>
<td>$140</td>
<td>44%</td>
</tr>
<tr>
<td>Normal growth</td>
<td>.50</td>
<td>110</td>
<td>14</td>
</tr>
<tr>
<td>Recession</td>
<td>.25</td>
<td>80</td>
<td>−16</td>
</tr>
</tbody>
</table>
How can we evaluate this probability distribution?

Throughout this book we will characterize probability distributions of rates of return in terms of their expected or mean return, $E(r)$, and their standard deviation, $\sigma$. 
The expected rate of return is a probability-weighted average of the rates of return in each scenario. Calling $p(s)$ the probability of each scenario and $r(s)$ the HPR in each scenario, where scenarios are labeled or “indexed” by the variable $s$, we may write the expected return as

$$E(r) = \sum_s p(s)r(s) \quad (5.1)$$
RISK AND RISK PREMIUMS

- Applying this formula to the data in Table 5.1, we find that the expected rate of return on the index fund is

\[
E(r) = (.25 \times 44\%) + (.5 \times 14\%) + [.25 \times (-16\%)]
= 14\%
\]
RISK AND RISK PREMIUMS

- The standard deviation of the rate of return ($\sigma$) is a measure of risk. It is defined as the square root of the variance, which in turn is the expected value of the squared deviations from the expected return.
- The higher the volatility in outcomes, the higher will be the average value of these squared deviations.
Symbolically,

\[ \sigma^2 = \sum_s p(s)[r(s) - E(r)]^2 \quad (5.2) \]

Therefore, in our example,

- \( \sigma^2 = .25(44 - 14)^2 + .5(14 - 14)^2 + .25(-16 - 14)^2 = 450 \)
- \( \sigma = \sqrt{450} = 21.21\% \)
Clearly, what would trouble potential investors in the index fund is the downside risk of a -16% rate of return, not the upside potential of a 44% rate of return. The standard deviation of the rate of return does not distinguish between these two: it treats both simply as deviations from the mean.
As long as the probability distribution is more or less symmetric about the mean, \( \sigma \) is an adequate measure of risk.

In the special case where we can assume that the probability distribution is normal—represented by the well-known bell-shaped curve—\( E(r) \) and \( \sigma \) are perfectly adequate to characterize the distribution.
Getting back to the example, how much, if anything, should you invest in the index fund?

First, you must ask how much of an expected reward is offered for the risk involved in investing money in stocks.
We measure the reward as the difference between the *expected* HPR on the index stock fund and the *risk-free rate*, that is, the rate you can earn by leaving money in risk-free assets such as T-bills, money market funds, or the bank. We call this difference the *risk premium* on common stocks.
If the risk-free rate in the example is 6% per year, and the expected index fund return is 14%, then the risk premium on stocks is 8% per year. The difference in any particular period between the actual rate of return on a risky asset and the risk-free rate is called excess return. Therefore, the risk premium is the expected excess return.
RISK AND RISK PREMIUMS

- The degree to which investors are willing to commit funds to stocks depends on risk aversion. Financial analysts generally assume investors are risk averse in the sense that, if the risk premium were zero, people would not be willing to invest any money in stocks.
In theory, then, there must always be a positive risk premium on stocks in order to induce risk-averse investors to hold the existing supply of stocks instead of placing all their money in risk-free assets.
Although this sample scenario analysis illustrates the concepts behind the quantification of risk and return, you may still wonder how to get a more realistic estimate of $E(r)$ and $\sigma$ for common stocks and other types of securities. Here history has insights to offer.
If the computation of standard deviation is based on historical data rather than forecasts of future scenarios as in equation 5.2, then the formula for historical variance becomes:

$$\sigma^2 = \frac{n}{n-1} \sum_{t=1}^{n} \frac{(r_t - \bar{r})^2}{n}$$
In the above equation, each year’s outcome \( (r_t) \) is taken as a possible scenario. Deviations are taken from the historical average, \( \bar{r} \). Each historical outcome is taken as equally likely and given a “probability” of 1/n. The equation is multiplied by \( n/(n – 1) \) to eliminate statistical bias for a “lost degree of freedom” in the estimate of variance.
Figure 5.3 gives a graphic representation of the relative variabilities of the annual HPR for three different asset classes. We have plotted the three time series on the same set of axes, each in a different color.
Figure 5.3  Rates of return on stocks, bonds, and T-bills, 1926–2002

Source: Prepared from data in Table 5.2.
The graph shows very clearly that the annual HPR on stocks is the most variable series. The standard deviation of large-stock returns has been 20.55% compared to 8.24% for long-term government bonds and 3.18% for bills.
Moreover, for large stocks, the arithmetic mean or average holding-period return is 12.04%; for long-term government bonds, 5.68%; and for bills, 3.82%.
Here is evidence of the risk-return trade-off that characterizes security markets: The markets with the highest average returns also are the most volatile, and vice versa.
Bills, Bonds, and Stocks, 1926-2002

- An all-stock portfolio with a standard deviation of 20.55% would represent a very volatile investment.
  - For example, if stock returns are normally distributed with a standard deviation of 20.55% and an expected rate of return of 12.04% (the historical average), in roughly one year out of three, returns will be less than -8.51% (12.04% - 20.55%) or greater than 32.59% (12.04% + 20.55%).
Figure 5.4 is a graph of the normal curve with mean 12.04% and standard deviation 20.55%. The graph shows the theoretical probability of rates of return within various ranges given these parameters.
Figure 5.4
The normal distribution

- 68.26%
- 95.44%
- 99.74%

-49.6  -29.1  -8.5  12.0  32.6  53.1  73.7
Figure 5.5 presents another view of the historical data, the actual frequency distribution of returns on various asset classes over the period 1926-2002.

- The first column of the figure gives the geometric averages of the historical rates of return on each asset class; this figure thus represents the compound rate of growth in the value of an investment in these assets.
Bills, Bonds, and Stocks, 1926-2002

- The second column shows the arithmetic averages that, absent additional information, might serve as forecasts of the future HPRs for these assets.
- The last column is the variability of asset returns, as measured by standard deviation.
Figure 5.5
Frequency distribution of annual HPRs, 1926–2002 (figures in percent)

<table>
<thead>
<tr>
<th>Series</th>
<th>Geometric Average</th>
<th>Arithmetic Average</th>
<th>Standard Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Small-Company Stocks*</td>
<td>11.64%</td>
<td>17.74%</td>
<td>39.30%</td>
</tr>
<tr>
<td>Large-Company Stocks</td>
<td>10.01</td>
<td>12.04</td>
<td>20.55</td>
</tr>
<tr>
<td>Long-Term Government Bonds</td>
<td>5.38</td>
<td>5.68</td>
<td>8.24</td>
</tr>
<tr>
<td>U.S. Treasury Bills</td>
<td>3.78</td>
<td>3.82</td>
<td>3.18</td>
</tr>
<tr>
<td>Inflation</td>
<td>3.05</td>
<td>3.14</td>
<td>4.37</td>
</tr>
</tbody>
</table>

*The 1933 small-company stock total return was 187.82% (not in diagram)
Source: Prepared from data in Table 5.2.
Again, the greater range of stock returns relative to bill or bond returns is obvious. The historical results are consistent with the risk–return trade-off: Riskier assets have provided higher expected returns, and historical risk premiums are considerable.
Figure 5.6 presents graphs of wealth indexes for investments in various asset classes over the period of 1926-2002. The plot for each asset class assumes you invest $1 at year-end 1925 and traces the value of your investment in following years.
Figure 5.6
Wealth indexes of investments in the U.S. capital markets, 1925–2002 (year-end 1925 = $1; logarithmic scale)

Source: Table 5.2.
The inflation plot demonstrates that to achieve the purchasing power represented by $1 in year-end 1925, one would require $10.11 at year-end 2002.
Bills, Bonds, and Stocks, 1926-2002

- One dollar continually invested in T-bills starting at year-end 1925 would have grown to $17.38 by year-end 2002, but provided only 1.72 times the original purchasing power \( (17.38/10.11 = 1.72) \).
That same dollar invested in large stocks would have grown to $1,548.32, providing 153 times the original purchasing power of the dollar invested ($1,548.32/10.11 = 153)—despite the great risk evident from sharp downturns during the period. Hence, the lesson of the past is that risk premiums can translate into vast increases in purchasing power over the long haul.
We should stress that variability of HPR in the past can be an unreliable guide to risk, at least in the case of the risk-free asset. For an investor with a holding period of 1 year, for example, a 1-year T-bill is a riskless investment, at least in terms of its nominal return, which is known with certainty.
However, the standard deviation of the 1-year T-bill rate estimated from historical data is not zero: This reflects variation over time in expected returns rather than fluctuations of actual returns around prior expectations.
The risk of cash flows of real assets reflects both *business risk* (profit fluctuations due to business conditions) and *financial risk* (increased profit fluctuations due to leverage).
This reminds us that an all-stock portfolio represents claims on leveraged corporations. Most corporations carry some debt, the service of which is a fixed cost. Greater fixed cost makes profits riskier; thus leverage increases equity risk.
REAL VERSUS NOMINAL RISK

- The distinction between the real and the nominal rate of return is crucial in making investment choices when investors are interested in the future purchasing power of their wealth. Thus a U.S. Treasury bond that offers a “risk-free” nominal rate of return is not truly a risk-free investment—it does not guarantee the future purchasing power of its cash flow.
EXAMPLE 5.2: Real versus Nominal Returns

- Consider a zero-coupon bond that pays $1,000 on a date 20 years from now but nothing in the interim. Suppose the price of the bond is $103.67, giving a nominal rate of return of 12% per year (since $103.67 \times 1.12^{20} = 1,000$). We compute in Table 5.3 the real annualized HPR for each inflation rate.
Table 5.3  Purchasing Power of $1,000 20 Years from Now and 20-Year Real Annualized HPR

<table>
<thead>
<tr>
<th>Assumed Annual Rate of Inflation</th>
<th>Number of Dollars Required 20 Years from Now to Buy What $1 Buys Today</th>
<th>Purchasing Power of $1,000 to Be Received in 20 Years</th>
<th>Annualized Real HPR</th>
</tr>
</thead>
<tbody>
<tr>
<td>4%</td>
<td>$2.19</td>
<td>$456.39</td>
<td>7.69%</td>
</tr>
<tr>
<td>6</td>
<td>3.21</td>
<td>311.80</td>
<td>5.66</td>
</tr>
<tr>
<td>8</td>
<td>4.66</td>
<td>214.55</td>
<td>3.70</td>
</tr>
<tr>
<td>10</td>
<td>6.73</td>
<td>148.64</td>
<td>1.82</td>
</tr>
<tr>
<td>12</td>
<td>9.65</td>
<td>103.67</td>
<td>0.00</td>
</tr>
</tbody>
</table>

Purchase price of bond is $103.67.
Nominal 20-year annualized HPR is 12% per year.
Purchasing power = $1,000/(1 + inflation rate)^20.
Real HPR, r, is computed from the following relationship: \[ r = \frac{1 + R}{1 + i} - 1 = \frac{1.12}{1 + i} - 1 \]
EXAMPLE 5.2: Real versus Nominal Returns

- Although some people see zero-coupon bonds as a convenient way for individuals to lock in attractive, risk-free, long-term interest rates, the evidence in Table 5.3 is rather discouraging about the value of $1,000 in 20 years in terms of today’s purchasing power.
EXAMPLE 5.2: Real versus Nominal Returns

- A revealing comparison is at a 12% rate of inflation. At that rate, Table 5.3 shows that the purchasing power of the $1,000 to be received in 20 years would be $103.67, the amount initially paid for the bond. The real HPR in these circumstances is zero.
EXAMPLE 5.2: Real versus Nominal Returns

- When the rate of inflation equals the nominal rate of interest, the price of goods increases just as fast as the money accumulated from the investment, and there is no growth in purchasing power.
EXAMPLE 5.2: Real versus Nominal Returns

- At an inflation rate of only 4% per year, however, the purchasing power of $1,000 will be $456.39 in terms of today’s prices; that is, the investment of $103.67 grows to a real value of $456.39, for a real 20-year annualized HPR of 7.69% per year.
EXAMPLE 5.2: Real versus Nominal Returns

- Again looking at Table 5.3, you can see that an investor expecting an inflation rate of 8% per year anticipates a real annualized HPR of 3.70%. If the actual rate of inflation turns out to be 10% per year, the resulting real HPR is only 1.82% per year. These differences show the important distinction between expected and actual inflation rates.
Even professional economic forecasters acknowledge that their inflation forecasts are hardly certain even for the next year, not to mention the next 20. When you look at an asset from the perspective of its future purchasing power, you can see that an asset that is risk-less in nominal terms can be very risky in real terms.
What does history teach us about the risk of a representative portfolio of stocks? The distribution of stock returns over the last 77 years may be a rough guide as to potential losses from future investments in the market. Let’s take a brief look at that distribution, focusing particularly on its “left tail,” representing investment losses.
The valuation of a stock is based on its (discounted) expected future cash flows, and the equilibrium price is set so as to yield a fair expected return, commensurate with its risk.
Deviations of the actual return from its expectation result from forecasting errors concerning the various economic factors that affect future cash flows.
RETURN DISTRIBUTIONS AND VALUE AT RISK

- It is natural to expect that the net effect of these many potential forecasting errors, and hence rates of return on stocks, will be at least approximately normally distributed.
The normal distribution has at least two attractive properties.

- First, it is symmetric and completely described by two parameters, its mean and standard deviation. This property implies that the risk of a normally distributed investment return is fully described by its standard deviation.
- Second, a weighted average of variables that are normally distributed also will be normally distributed.
Therefore, when individual stock returns are normally distributed, the return on any portfolio combining any set of stocks will also be normally distributed and its standard deviation will fully convey its risk. For these reasons, it is useful to know whether the assumption of normality is warranted. Figure 5.5 is derived from raw annual returns and is too crude to make this determination.
Figure 5.7 compares the cumulative probability distributions of continuously compounded monthly returns on large and small stocks with those of normal distributions with matching means and standard deviations.
Figure 5.7
Actual versus normal cumulative frequencies of monthly returns for (a) large stocks and (b) small stocks, 1926–2002 (returns are annualized and expressed as continuously compounded percentage rates)
While the cumulative distributions of both large and small stocks roughly match the normal distribution, we can see that extreme values are more frequent than suggested by the normal. Moreover, these extreme values appear to be more frequent at the negative end of the distribution.
We would like a measure that reveals whether extreme negative values are too large and frequent to be adequately characterized by the normal distribution. One approach is to compute the standard deviation separately for the negative deviations. This statistic is called lower partial standard deviation (LPSD).
If the LPSD is large relative to the overall standard deviation, it should replace the standard deviation as a measure of risk, since investors presumably think about risk as the possibility of large negative outcomes.
Another way to describe the negative extreme values is to quantify the asymmetry of the distribution. This we do by averaging the cubed (instead of squared) deviations from the mean. Just as the variance is called the second moment around the mean, the average cubed deviation is called the *third moment* of the distribution.
The cubed deviations preserve sign, which allows us to distinguish good from bad outcomes. Moreover, since cubing exaggerates larger deviations, the sign and magnitude of the average cubed deviation reveal the direction and extent of the asymmetry.
To scale the third moment (its unit is percent cubed), we divide it by the cubed standard deviation. The resulting statistic is called the skew of the distribution, and skewness (偏態) is a measure of asymmetry.
Table 5.4 compares the lower partial standard deviations of large and small stocks to the overall standard deviations.
### Table 5.4 Deviations from Normality and VaR
(Continuously Compounded Monthly Returns, 1926–2002)

<table>
<thead>
<tr>
<th></th>
<th>Large Stocks</th>
<th>Small Stocks</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>9.54%</td>
<td>11.01%</td>
</tr>
<tr>
<td>Standard deviation</td>
<td>19.83%</td>
<td>33.25%</td>
</tr>
<tr>
<td>LPSD</td>
<td>28.36%</td>
<td>39.90%</td>
</tr>
<tr>
<td>Skew</td>
<td>-0.98</td>
<td>-0.66</td>
</tr>
<tr>
<td>VaR Normal</td>
<td>-23.18%</td>
<td>-43.85%</td>
</tr>
<tr>
<td>VaR Actual</td>
<td>-30.07%</td>
<td>-62.04%</td>
</tr>
</tbody>
</table>
Indeed, the LPSD is much larger than the overall standard deviation. The next line indicates that both distributions are negatively skewed.
Professional investors extensively use a risk measure that highlights the potential loss from extreme negative returns, called value at risk (風險值或涉險值), denoted by VaR.
The VaR is another name for the quantile of a distribution. The quantile \( q \) of a distribution is the value below which lie \( q\% \) of the values. Thus the median of the distribution is the 50% quantile.
Practitioners commonly call the 5% quantile the VaR of the distribution. It tells us that, with a probability of 5%, we can expect a loss equal to or greater than the VaR.
For a normal distribution, which is completely described by its mean and standard deviation, the VaR always lies 1.65 standard deviations below the mean, and thus, while it may be a convenient benchmark, it adds no information about risk.
But if the distribution is not adequately described by the normal, the VaR does give useful information about the magnitude of loss we can expect in a “bad” (i.e., 5% quantile) scenario.
Table 5.4 shows the 5% VaR based on normal distributions with the same means and standard deviations as large and small stocks.

- Actual VaR based on historic returns show significantly greater losses for large stocks (-30% vs. -23%) and for small stocks (-62% vs. -44%).
Table 5.4  Deviations from Normality and VaR  
(Continuously Compounded Monthly Returns, 1926–2002)

<table>
<thead>
<tr>
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RETURN DISTRIBUTIONS AND VALUE AT RISK

- One can compute the VaR corresponding to other quantiles, for example a 1% VaR. The VaR has become widely used by portfolio managers and regulators as a measure of potential losses.
A GLOBAL VIEW OF THE HISTORICAL RECORD

- As financial markets around the world grow and become more regulated and transparent, U.S. investors look to improve diversification by investing internationally. Foreign investors that traditionally used U.S. financial markets as a safe haven to supplement home-country investments also seek international diversification to reduce risk.
A GLOBAL VIEW OF THE HISTORICAL RECORD

- The question arises as to how historical U.S. experience compares with that of stock markets around the world.
Figure 5.8 shows a century-long history (1900-2000) of average nominal and real returns in stock markets of 16 developed countries.
Figure 5.8 Nominal and real equity returns around the world, 1900–2000

A GLOBAL VIEW OF THE HISTORICAL RECORD

- We find the United States in fourth place in terms of real, average returns, behind Sweden, Australia, and South Africa.
A GLOBAL VIEW OF THE HISTORICAL RECORD

- Figure 5.9 shows the standard deviations of real stock and bond returns for these same countries.
Figure 5.9  Standard deviations of real equity and bond returns around the world, 1900–2000

A GLOBAL VIEW OF THE HISTORICAL RECORD

- We find the United States tied with four other countries for third place in terms of lowest standard deviation of real stock returns.

- So the United States has done well, but not abnormally so, compared with these countries.
One interesting feature of these figures is that the countries with the worst results, measured by the ratio of average real returns to standard deviation, are Italy, Belgium, Germany, and Japan—the countries most devastated by World War II.
A GLOBAL VIEW OF THE HISTORICAL RECORD

- The top-performing countries are Australia, Canada, and the United States, the countries least devastated by the wars of the twentieth century.
Another, perhaps more telling feature, is the insignificant difference between the real returns in the different countries. The difference between the highest average real rate (Sweden, at 7.6%) from the average return across the 16 countries (5.1%) is 2.5%. Similarly, the difference between the average and the lowest country return (Belgium, at 2.5%) is 2.6%.
Using the average standard deviation of 23%, the $t$-statistic for a difference of 2.6% with 100 observations is

$$t\text{-statistic} = \frac{\text{Difference in mean}}{\text{Standard deviation}/\sqrt{n}} = \frac{2.6}{23 / \sqrt{100}} = 1.3$$

which is far below conventional levels of statistical significance.
A GLOBAL VIEW OF THE HISTORICAL RECORD

- We conclude that the U.S. experience cannot be dismissed as an outlier case. Hence, using the U.S. stock market as a yardstick for return characteristics may be reasonable.
FORECASTS FOR THE LONG HAUL

- While portfolio managers struggle daily to adjust their portfolios to changing expectations, individual investors are more concerned with long-term expectations.
A middle-aged investor at the age of 40 is typically saving largely for retirement with a horizon of some 25 years. With a $100,000 investment, a 1% increase in the average real rate, from 6% to 7%, would increase the real retirement fund in 25 years from $100,000 \times 1.06^{25} = $429,187 to $100,000 \times 1.07^{25} = $542,743, a difference of $113,556 in real-consumption dollars.
FORECASTS FOR THE LONG HAUL

- You can see why a reliable long-term return forecast is crucial to the determination of how much to save and how much of the savings to invest in risky assets for the risk premium they offer.
FORECASTS FOR THE LONG HAUL

- The Japanese experience in the 1990s provides another idea of the stakes involved. In 1990, the Nikkei 225 (an index of Japanese stocks akin to the S&P 500 in the United States) stood at just about 40,000. By 2003, that index stood at about 8,000, a nominal loss of 80% over 13 years and an even greater loss in real terms.
FORECASTS FOR THE LONG HAUL

- A Japanese investor who invested in stocks at the age 50 in 1990 would have found himself in 2003, only a few years before retirement, with less than 20% of his 1990 savings. For such investors, the rosy long-term history of stock market investments is cold comfort.
FORECASTS FOR THE LONG HAUL

- These days, practitioners and scholars are debating whether the historical U.S. average risk-premium of large stocks over T-bills of 8.22% (12.04% – 3.82%; see Table 5.2), is a reasonable forecast for the long term.
FORECASTS FOR THE LONG HAUL

- This debate centers around two questions:
  - First, do economic factors that prevailed over that historic period (1926-2002) adequately represent those that may prevail over the forecasting horizon?
  - Second, is the arithmetic average from the available history a good yardstick for long-term forecasts?
Fama and French note that the risk premium on equities over the period 1872-1949 was 4.62%, much lower than the 8.41% premium realized over the period 1950-1999. They argue that returns over the later half of the twentieth century were largely driven by unexpected capital gains that cannot be expected to prevail in the next century.
FORECASTS FOR THE LONG Haul

- Figure 5.5 shows that the large-stock risk premium as measured by geometric averages over the period 1926-2002 is much lower, at 6.23% (i.e., 10.01% – 3.78%), than the 8.22% premium based on the arithmetic average. This difference alone would take away about half of the range of the debate. So which average should we use?
The use of the arithmetic average to forecast future rates of return derives from the statistical property that it is unbiased. This desirable property has made arithmetic averages the norm for estimating expected holding-period rates of return. However, for forecasts of cumulative returns over long horizons, the arithmetic average is inadequate.
Jacquier, Kane, and Marcus show that the correct forecast of total returns over long horizons requires compounding at a weight applied to the geometric average equals the ratio of the length of the forecast horizon to the length of the estimation period.
FORECASTS FOR THE LONG HAUL

- For example, if we wish to forecast the cumulative return for a 25-year horizon from a 77-year history, an unbiased estimate would be to compound at a rate of

  Geometric average \times \left( \frac{25}{77} \right) +
  Arithmetic average \times \left[ \frac{(77 - 25)}{77} \right]
FORECASTS FOR THE LONG HAUL

- This correction would take about 0.7% off the historical arithmetic average risk premium on large stocks and about 2% off the arithmetic average of small stocks.
FORECASTS FOR THE LONG HAUL

- A forecast for the next 77 years would require compounding at only the geometric average, and for longer horizons at an even lower number.

- The forecast horizons that are relevant for current middle-aged investors must, however, be resolved primarily on economic rather than statistical grounds.