1. (65%)
(a) The probability density function for a continuous random variable $X$ is

$$f(x) = \begin{cases} 
  kx(1-x), & 0 < x < 1 \\
  0, & \text{otherwise}
\end{cases}$$

Please find (a) $k$ (b) $P\left(\frac{1}{4} \leq X \right)$ (c) $E(X)$

(b) The time between breakdown of a new automated production process is exponentially distributed with mean 16 (hour/breakdown).

(i) What is the probability that the process can run 2 days shift without a breakdown?
(ii) What is the probability that there are 3 breakdowns within 8 hours?
(iii) What is the probability that a breakdown occurs between 8 and 16 hours?

(c) A retailer of electronic equipment received six VCRs from the manufacturer. Three of the VCRs were damaged in the shipment. The retailer sold two VCRs to two customers.

(i) What is the probability that both customers received damaged VCRs?
(ii) What is the probability that one of the two customers received a defective VCR?

(d) Given that $Z$ is a standard normal random variable and $X$ is a normal random variable with mean 5 and standard deviation 2.

(i) $P(-1.57 \leq Z \leq 0.49)$
(ii) $P(0.44 \leq Z)$
(iii) $P(1 \leq X \leq 7)$
(iv) $P(-c \leq Z \leq c) = 0.2052$. Find $c$.
(v) $P(3 \leq X \leq c) = 0.7745$. Find $c$.

2. (40%)
(a) Students of a large university spend an average of $5 a day on lunch. The standard deviation of the expenditure is $3. A simple random sample of 36 students is taken.

(i) What is the probability that the sample mean will be at least $4? 
(ii) What is the probability that the sample mean will be greater than $5.90? 
(iii) How large of a sample needs to be taken to provide a 0.4949 probability that the sample mean will be between $5 and $6?

(b) A population proportion is 0.4. A simple random sample of size 200 will be taken and the sample proportion $\bar{p}$ will be used to estimate the population
proportion.

(i) What is the probability that the sample proportion will be within ±0.03 of the population proportion?

(ii) What is the probability that the sample proportion will be within ±0.05 of the population proportion?

3. (25%)

(a) Suppose we have a population of 27 elements

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Suppose the first row of the table of random number is

63271 59986 71744 51102 15141 80714 58683 93108 13554 79945

Please use simple random sampling to obtain

(i) a sample of 5 elements.

(ii) the sample mean, 60’th percentile and interquartile range based on (i).

(b) A statistics teacher has noted from past experience that a student who follows a program of studying two hours for each hour in class has a probability of 0.9 of getting a grade of C or better, while a student who does not follow a regular study program has a probability of 0.2 of getting a C or better. It is known that 70% of the students follow the study program. Find the probability that if a student who has earned a C or better grade, he/she followed the program.