Chapter 5 Discrete Probability Distributions

5.1. Random variable:

Example 1:
Suppose we gamble in a casino and the possible result is as follows.

<table>
<thead>
<tr>
<th>Outcome</th>
<th>Token ($X$)</th>
<th>Money ($Y$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Win</td>
<td>3</td>
<td>30</td>
</tr>
<tr>
<td>Lose</td>
<td>-4</td>
<td>-40</td>
</tr>
<tr>
<td>Tie</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

In this example, the sample space is $S = \{\text{Win}, \text{Lose}, \text{Tie}\}$, containing 3 outcomes.

$X$ is the quantity representing the token obtained or lose under different result while $Y$ is the one representing the money obtained or lost. In the above example, $X$ and $Y$ can provide a numerical summary corresponding to the experimental outcome. A formal definition for these numerical quantities is in the following.

Definition (random variable): A random variable is a numerical description of the outcome of an experiment.

Note: A random variable $X$ is defined as a real-valued function on the sample space, i.e.,

$$X: S \rightarrow R.$$ 

Example 1 (continue):
In the previous example,

$X$: the random variable representing the token obtained or lose corresponding to different outcomes.

$Y$: the random variable representing the money obtained or lose corresponding to different outcomes.

$X$ has 3 possible values corresponding to 3 outcomes

$$X(\text{Win}) = 3, X(\text{Lose}) = -4, X(\text{Tie}) = 0.$$
\( Y \) has 3 possible values corresponding to 3 outcomes
\[ \Rightarrow Y(Win) = 30, Y(Lose) = -40, Y(Tie) = 0. \]

Note that \( Y = 10X \) since
\[ Y(Win) = 30 = 10X(Win), Y(Lose) = -40 = 10X(Lose), \]
\[ Y(Tie) = 0 = 10X(Tie) \]
That is, \( Y \) is 10 times of \( X \) under all possible experimental outcomes.

There are two types of random variables. They are:

**Discrete random variable:** A quantity assumes either a finite number of values or an infinite sequence of values, such as \( 0, 1, 2, \ldots \).

**Continuous random variable:** A quantity assumes any numerical value in an interval or collection of intervals, such as time, weight, distance, and temperature.

**Example 2:**
Let the sample space
\[ S = \{ x \text{ hour} | x \text{ is the delay time for a flight}, 0 \leq x \leq 1 \}. \]
Let \( X \) be the random variable representing the delay flight time, defined as
\[ X(x \text{ hour}) = x, 0 \leq x \leq 1. \]
For example, \( X = 0.5 \) corresponds to the outcome that the flight time is 0.5 hour (30 minutes) late.