CHAPTER 14

BOND PRICES AND YIELDS
BOND PRICES AND YIELDS

- Bond Characteristics
- Bond Pricing
- Bond Yields
- Bond Prices over Time
- Default Risk and Bond Pricing
A **bond** is a security that is issued in connection with a borrowing arrangement. The borrower issues (i.e., sells) a bond to the lender for some amount of cash; the bond is the ‘‘IOU’’ of the borrower. The arrangement obligates the issuer to make specified payments to the bondholder dates.
A typical coupon bond obligates the issuer to make semiannual payments of interest to the bondholder for the life of the bond. These are called *coupon payments* because in precomputer days, most bonds had coupons that investors would clip off and mail to the issuer of the bond to claim the interest payment.
BOND CHARACTERISTICS

- When the bond matures, the issuer repays the debt by paying the bondholder the bond’s par value (equivalently, its face value).
- The couple rate of the bond serves to determine the interest payment: The annual payment is the coupon rate times the bond’s par value.
The coupon rate, maturity date, and par value of the bond are part of the bond indenture, which is the contract between the issuer and the bondholder.
BOND CHARACTERISTICS

- To illustrate, a bond with par value of $1,000 and coupon rate of 8% might be sold to a buyer for $1,000.
  - The bondholder is then entitled to a payment of 8% of $1,000, or $80 per year, for the stated life of the bond, say 30 years. The $80 payment typically comes in two semiannual installments of $40 each.
  - At the end of the 30-year life of the bond, the issuer also pays the $1,000 par value to the bondholder.
BOND CHARACTERISTICS

- Bonds usually are issued with coupon rates set high enough to induce investors to pay par value to buy the bonds. Sometimes, however, **zero-coupon bonds** are issued that make no coupon payments. In this case, investors receive par value at the maturity date but receive no interest payments until then: The bond has a coupon rate of zero.
BOND CHARACTERISTICS

- Zero-coupon bonds are issued at prices considerably below par value, and the investor’s return comes solely from the difference between issue price and the payment of par value at maturity.
Treasury Bonds and Note

- Treasury note maturities range up to 10 years, while Treasury bonds with maturities ranging from 10 to 30 years appear in the figure. Both bonds and notes are issued in denominations of $1,000 or more. Both make semiannual coupon payments.
Treasury Bonds and Note

- Aside from their differing initial maturities, the only major distinction between T-notes and T-bonds is that in the past, some T-bonds were callable for a given period, usually during the last 5 years of the bond’s life. The call provision gives the Treasury the right to repurchase the bond at par value during the call period. The Treasury no longer issues callable bonds.
Figure 14.1 is an excerpt from the listing of Treasury issues in *The Wall Street Journal*. The highlighted bond in Figure 14.1 matures in February 2012. Its coupon rate is 4.875%. Par value is $1,000; thus the bond pays interest of $48.75 per year in two semiannual payments of $24.375. Payments are made in February and August of each year.
# Treasury Bonds and Note

Figure 14.1 Listing of Treasury issues

<table>
<thead>
<tr>
<th>RATE</th>
<th>MATURITY</th>
<th>BID</th>
<th>ASKED</th>
<th>CHG</th>
<th>ASK YLD</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.875</td>
<td>Feb 12n</td>
<td>107:20</td>
<td>107:21</td>
<td>-20</td>
<td>3.86</td>
</tr>
</tbody>
</table>
Treasury Bonds and Note

- The bid and asked prices are quoted in points plus fractions of 1/32 of a point (the numbers after the colons are the fractions of a point). Although bonds are sold in denominations of $1,000 par value, the prices are quoted as a percentage of par value.
Treasury Bonds and Note

Therefore, the bid price of the bond is 107:20
= 107(20/32) = 107.625 of par value, or
$1076.25, whereas the asked price is
107(21/32)% of par, or $107.5625.
Treasury Bonds and Note

- Recall that the bid price is the price at which one can sell the bond to a dealer. The asked price, which is slightly higher, is the price at which one can buy the bond from a dealer.
Treasury Bonds and Note

- The last column, labeled “‘Ask Yld,’” is the yield to maturity on the bond based on the asked price. The yield to maturity is a measure of the average rate of return to an investor who purchases the bond for the asked price and holds it until its maturity date. We will have much to say about yield to maturity below.
Accrued Interest and Quoted Bond Prices

- The bond prices that you see quoted in the financial pages are not actually the prices that investors pay for the bond. This is because the quoted price does not include the interest that accrues between coupon payment dates.
Accrued Interest and Quoted Bond Prices

- If a bond is purchased between coupon payments, the buyer must pay the seller for accrued interest, the prorated share of the upcoming semiannual coupon.
  - For example, if 40 days have passed since the last coupon payment, and there are 182 days in the semiannual coupon period, the seller is entitled to a payment of accrued interest of 40/182 of the semiannual coupon. The sale, or *invoice price*, of the bond would equal the stated price plus the accrued interest.
Accrued Interest and Quoted Bond Prices

- In general, the formula for the amount of accrued interest between two dates is

\[
\text{Accrued interest} = \left(\frac{\text{Annual coupon payment}}{2}\right) \times \left(\frac{\text{Days since last coupon payment}}{\text{Days separating coupon payments}}\right)
\]
**EXAPMLE 14.1: Accrued interest**

- Suppose that the coupon rate is 8%. Then the annual coupon is $80 and the semiannual coupon payment is $40. Because 40 days have passed since the last coupon payment, the accrued interest on the bond is $40 \times \left(\frac{40}{182}\right) = $8.79. If the quoted price of the bond is $990, then the invoice price will be $990 + $8.79 = $998.79.
The practice of quoting bond prices net of accrued interest explains why the price of a maturing bond is listed at $1,000 rather than $1,000 plus one coupon payment.

A purchaser of an 8% coupon bond 1 day before the bond’s maturity would receive $1,040 (par value plus semiannual interest) on the following day and so should be willing to pay a total price of $1,040 for the bond. The bond price is quoted net of accrued interest in the financial pages and thus appears as $1,000.
Corporate Bonds

- Like the government, corporations borrow money by issuing bonds. Although some bonds trade on a formal exchange operated by the New York Stock Exchange, most bonds are traded over the counter in a network of bond dealers linked by a computer quotation system.
  - See Chapter 3 for a comparison of exchange versus OTC trading.
Corporate Bonds

- In practice, the bond market can be quite “thin,” in that there are few investors interested in trading a particular issue at any particular time.
Corporate Bonds

- Figure 14.2 is a sample of corporate bond listing in *The Wall Street Journal*, which lists only the most actively traded corporate bonds.
  - The highlighted Ford bond in Figure 14.2 has coupon rate of 7.45% and matures on July 16, 2031.
  - The last price at which the bond traded was 85.417% of par value, or $854.17, and the yield to maturity based on this price was 8.868%.
### Corporate Bonds

**Monday, August 4, 2003**

Forty most active fixed-coupon corporate bonds

<table>
<thead>
<tr>
<th>COMPANY (TICKER)</th>
<th>COUPON</th>
<th>MATURITY</th>
<th>LAST PRICE</th>
<th>LAST YIELD</th>
<th>EST SPREAD</th>
<th>EST $ VOL (100's)</th>
</tr>
</thead>
<tbody>
<tr>
<td>General Motors (GM)</td>
<td>8.375</td>
<td>Jul 15, 2033</td>
<td>94.229</td>
<td>8.929</td>
<td>368</td>
<td>138,007</td>
</tr>
<tr>
<td>Liberty Media (L)</td>
<td>5.700</td>
<td>May 15, 2013</td>
<td>95.615</td>
<td>6.306</td>
<td>199</td>
<td>131,245</td>
</tr>
<tr>
<td>Walt Disney (DIS)-c</td>
<td>2.125</td>
<td>Apr 15, 2023</td>
<td>104.590</td>
<td>1.117</td>
<td>n.a.</td>
<td>95,518</td>
</tr>
<tr>
<td>General Motors Acceptance (GMAC)</td>
<td>8.000</td>
<td>Nov 01, 2031</td>
<td>92.866</td>
<td>8.679</td>
<td>342</td>
<td>50,355</td>
</tr>
<tr>
<td>AT&amp;T Wireless Services (AWE)</td>
<td>8.125</td>
<td>May 01, 2012</td>
<td>114.500</td>
<td>5.968</td>
<td>164</td>
<td>49,789</td>
</tr>
<tr>
<td>General Motors (GM)</td>
<td>7.125</td>
<td>Jul 15, 2013</td>
<td>98.349</td>
<td>7.360</td>
<td>308</td>
<td>48,806</td>
</tr>
<tr>
<td>Ford Motor (F)</td>
<td>7.450</td>
<td>Jul 16, 2031</td>
<td>95.417</td>
<td>8.868</td>
<td>362</td>
<td>47,835</td>
</tr>
<tr>
<td>Bank One (ONE)</td>
<td>7.625</td>
<td>Aug 01, 2005</td>
<td>110.622</td>
<td>2.127</td>
<td>44</td>
<td>47,750</td>
</tr>
<tr>
<td>General Motors (GM)</td>
<td>8.250</td>
<td>Jul 15, 2023</td>
<td>94.719</td>
<td>8.815</td>
<td>350</td>
<td>46,444</td>
</tr>
<tr>
<td>Ford Motor Credit (F)</td>
<td>7.500</td>
<td>Mar 15, 2005</td>
<td>105.822</td>
<td>3.727</td>
<td>205</td>
<td>45,866</td>
</tr>
<tr>
<td>GIT Group (GIT)</td>
<td>4.000</td>
<td>May 08, 2008</td>
<td>99.997</td>
<td>4.007</td>
<td>90</td>
<td>42,215</td>
</tr>
<tr>
<td>Aven Products (AVP)</td>
<td>7.150</td>
<td>Nov 15, 2009</td>
<td>115.371</td>
<td>4.324</td>
<td>122</td>
<td>30,000</td>
</tr>
</tbody>
</table>

Volume represents total volume for each issue; price/yield data are for trades of $1 million and greater. Estimated spreads, in basis points (100 basis points is one percentage point), over the 2, 3, 5, 10 or 30-year Treasury note/bond. 2-year: 1.500 07/05; 3-year: 2.000 05/06; 5-year: 2.625 05/08; 10-year: 3.425 05/13; 30-year: 5.375 02/31. Comparable U.S. Treasury issue. c-Convertible bond.

Source: MarketAxess Corporate BondTicker

Corporate Bonds

- The next column shows the difference or “spread” between the yield to maturity on this bond, and the yield to maturity of a comparable-maturity U.S. Treasury bond. The spread is given in terms of basis points (1 basis point is .01%). Therefore, the spread for the Ford bond is 362 basis points, or 3.62%.
Corporate Bonds

- The maturity of the Treasury bond against which the Ford bond is compared is 30 years. The yield spread reflects the default or credit risk of the Ford bond. We will have much to say about the determinants of credit risk later in the chapter.
Corporate Bonds

- Bonds issued in the United States today are *registered*, meaning that the issuing firm keeps records of the owner of the bond and can mail interest checks to the owner. Registration of bonds is helpful to tax authorities in the enforcement of tax collection.
Corporate Bonds

- *Bearer bonds* are those traded without any record of ownership. The investor’s physical possession of the bond certificate is the only evidence of ownership. These are now rare in the United States, but less rare in Europe.
Call Provisions on Corporate Bonds

- Although the Treasury no longer issues callable bonds, some corporate bonds are issued with call provision allowing the issuer to repurchase the bond at a specified call price before the maturity date.
Call Provisions on Corporate Bonds

- For example, if a company issues a bond with a high coupon rate when market interest rates are high, and interest rates later fall, the firm might like to retire the high-coupon debt and issue new bonds at a lower coupon rate to reduce interest payment. This is called *refunding*. 
Callable bonds typically come with a period of call protection, an initial time during which the bonds are not callable. Such bonds are referred to as deferred callable bonds.
Call Provisions on Corporate Bonds

- The option to call the bond is valuable to the firm, allowing it to buy back the bonds and refinance at lower interest rates when market rates fall. Of course, the firm’s benefit is the bondholder’s burden.
Call Provisions on Corporate Bonds

- Holders of called bonds forfeit their bonds for the call price, thereby giving up the prospect of an attractive coupon rate on their original investment. To compensate investors for this risk, callable bonds are issued with higher coupons and promised yields to maturity than noncallable bonds.
Convertible Bonds

- **Convertible bonds** give bondholders an option to exchange each bond for a specified number of shares of common stock of the firm. The *conversion ratio* is the number of shares for which each bond may be exchanged.
Suppose a convertible bond is issued at par value of $1,000 and is convertible into 40 shares of a firm’s stock. The current stock price is $20 per share, so the option to convert is not profitable now. Should the stock price later rise to $30, however, each bond may be converted profitably into $1,200 worth of stock.
Convertible Bonds

- The *market conversion value* is the current value of the shares for which the bonds may be exchanged. At the $20 stock price, for example, the bond’s conversion value is $800. If the bond were selling currently for $950, its premium would be $150.
Convertible Bonds

- Convertible bondholders benefit from price appreciation of the company’s stock. Again, this benefit comes at a price: Convertible bonds offer lower coupon rates and stated or promised yields to maturity than do nonconvertible bonds. However, the actual return on the convertible bond may exceed the stated yield to maturity if the option to convert becomes profitable.
Puttable Bonds

- While the callable bond gives the issuer the option to extend or retire the bond at the call date, the *extendable* or *put bond* gives this option to the bondholder.
Puttable Bonds

- If the bond’s coupon rate exceeds current market yields, for instance, the bondholder will choose to extend the bond’s life. If the bond’s coupon rate is too low, it will be optimal not to extend; the bondholder instead reclaims principal, which can be invested at current yields.
Floating-Rate Bonds

- **Floating-rate bonds** make interest payments that are tied to some measure of current market rates.
  - For example, the rate might be adjusted annually to the current T-bill rate plus 2%. If the 1-year T-bill rate at the adjustment date is 4%, the bond’s coupon rate over the next year would then be 6%. This arrangement means that the bond always pays approximately current market rates.
Floating-Rate Bonds

- The major risk involved in floaters has to do with changes in the firm’s financial strength. The yield spread is fixed over the life of the security, which may be many years.
If the financial health of the firm deteriorates, then a greater yield premium would be called for than is offered by the security. In this case, the price of the bond would fall. Although the coupon rate on floaters adjusts to changes in the general level of market interest rates, it does not adjust to changes in the financial condition of the firm.
Preferred Stock

- Although preferred stock strictly speaking is considered to be equity, it often is included in the fixed-income universe. This is because, like bonds, preferred stock promises to pay a specified stream of dividends.
Preferred Stock

- However, unlike bonds, the failure to pay the promised dividend does not result in corporate bankruptcy. Instead, the dividends owed simply cumulate, and the common stockholders may not receive any dividends until the preferred stockholders have been paid in full.
Preferred Stock

- In the event of bankruptcy, preferred stockholders’ claims to the firm’s assets have lower priority than those of bondholders, but higher priority than those of common stockholders.
Preferred Stock

- Preferred stock commonly pays a fixed dividend. Therefore, it is in effect a perpetuity, providing a level cash flow indefinitely.
Preferred Stock

- In the last two decades, however, adjustable or floating-rate preferred stock has become popular, in some years accounting for about half of new issues. Floating-rate preferred stock is much like floating-rate bonds. The dividend rate is linked to a measure of current market interest rates and is adjusted at regular intervals.
Preferred Stock

- Unlike interest payments on bonds, dividends on preferred stock are not considered tax-deductible expenses to the firm. This reduces their attractiveness as a source of capital to issuing firms.
Preferred Stock

- On the other hand, there is an offsetting tax advantage to preferred stock. When one corporation buys the preferred stock of another corporation, it pays taxes on only 30% of the dividends received.
Preferred Stock

- For example, if the firm’s tax bracket is 35%, and it receives $10,000 in preferred dividend payments, it will pay taxes on only $3,000 of that income: Total taxes owned on the income will be .35 × $3,000 = $1,050.
The firm’s effective tax rate on preferred dividends is therefore only $0.30 \times 35\% = 10.5\%$. Given this tax rule, it is not surprising that most preferred stock is held by corporations.
Preferred Stock

- Preferred stock rarely gives its holders full voting privileges in the firm. However, if the preferred dividend is skipped, the preferred stockholders will then be provided some voting power.
Other Issuers

- There are, of course, several issuers of bonds in addition to the Treasury and private corporations. For example, state and local governments issue municipal bonds. The outstanding feature of these is that interest payments are tax-free. We examined municipal bonds and the value of the tax exemption in Chapter 2.
Other Issuers

- Government agencies such as the Federal Home Loan Bank Board, the Farm Credit agencies, and the mortgage pass-through agencies Ginnie Mae, Fannie Mae, and Freddie Mac, also issue considerable amounts of bonds. These too were reviewed in Chapter 2.
International Bonds

- International bonds are commonly divided into two categories, *foreign bonds* and *Eurobonds*. 
International Bonds

- **Foreign bonds** are issued by a borrower from a country other than the one in which the bond is sold. The bond is denominated in the currency of the country in which it is marketed.
  
  - For example, if a German firm sells a dollar-denominated bond in the United States, the bond is considered a foreign bond.
International Bonds

- These bonds are given colorful names based on the countries in which they are marketed.
  - Foreign bonds sold in the United States are called *Yankee bonds*. Like other bonds sold in the United States, they are registered with the Securities and Exchange Commission.
  - Yen-denominated bonds sold in Japan by non-Japanese issues are called *Samurai bonds*.
  - British pound-denominated foreign bonds sold in the United Kingdom are called *bulldog bonds*. 
International Bonds

- **Eurobonds** are bonds issued in the currency of one country but sold in other national markets.
  - For example, the Eurodollar market refers to dollar-denominated bonds sold outside the United States (not just in Europe), although London is the largest market for Eurodollar bonds.
International Bonds

- Because the Eurodollar market falls outside U.S. jurisdiction, these bonds are not regulated by U.S. federal agencies.
- Similarly, Euroyen bonds are yen-denominated bonds selling outside Japan, Eurosterling bonds are pound-denominated Eurobonds selling outside the United Kingdom, and so on.
Innovation in the Bond Market

- Issues constantly develop innovative bonds with unusual features; these issues illustrate that bond design can be extremely flexible. Here are examples of some novel bonds. They should give you a sense of the potential variety in security design.
Inverse Floaters

- These are similar to the floating-rate bonds we described earlier, except that the coupon rate on these bonds *falls* when the general level of interest rates rises.
Inverse Floaters

- Investors in these bonds suffer doubly when rates rise. Not only does the present value of each dollar of cash flow from the bond fall as the discount rate rises, but the level of those cash flows falls as well. Of course, investors in these bonds benefit doubly when rates fall.
Asset-Backed Bonds

- Walt Disney has issued bonds with coupon rates tied to the financial performance of several of its films. Similarly, “David Bowie bonds” have been issued with payments that will be tied to royalties on some of his albums. These are examples of asset-backed securities.
Asset-Backed Bonds

- The income from a specified group of assets is used to service the debt. More conventional asset-backed securities are mortgage-backed securities or securities backed by auto or credit card loans, as we discussed in Chapter 2.
Catastrophe Bonds

- Electrolux once issued a bond with a final payment that depended on whether there had been an earthquake in Japan. Winterthur has issued a bond whose payments depend on whether there has been a severe hailstorm in Switzerland.
Catastrophe Bonds

- These bonds are a way to transfer “catastrophe risk” from the firm to the capital markets. They represent a novel way of obtaining insurance from the capital markets against specified disasters. Investors in these bonds receive compensation for taking on the risk in the form of higher coupon rates.
Indexed Bonds

- Indexed bonds make payments that are tied to a general price index or the price of a particular commodity.
  - For example, Mexico has issued 20-year bonds with payments that depend on the price of oil.
  - Some bonds are indexed to the general price level.
  - The United States Treasury started issuing such inflation-indexed bonds in January 1997. They are called Treasury Inflation Protected Securities (TIPS).
Indexed Bonds

- By tying the par value of the bond to the general level of prices, coupon payments as well as the final repayment of par value on these bonds increase in direct proportion to the consumer price index. Therefore, the interest rate on these bonds is a risk-free real rate.
Indexed Bonds

- To illustrate how TIPS work. Consider a newly issued bond with a 3-year maturity, par value of $1,000, and a coupon rate of 4%. For simplicity, we will assume the bond makes annual coupon payments. Assume that inflation turns out to be 2%, 3%, and 1% in the next 3 years. Table 14.1 shows how the bond cash flows will be calculated.
<table>
<thead>
<tr>
<th>Time</th>
<th>Inflation in Year Just Ended</th>
<th>Par Value</th>
<th>Coupon Payment</th>
<th>Principal Repayment</th>
<th>Total Payment</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td></td>
<td>$1,000.00</td>
<td>$1,000.00</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>2%</td>
<td>1,020.00</td>
<td>$40.80</td>
<td>$0</td>
<td>$40.80</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
<td>1,050.60</td>
<td>42.02</td>
<td>0</td>
<td>42.02</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>1,061.11</td>
<td>42.44</td>
<td>1,061.11</td>
<td>1,103.55</td>
</tr>
</tbody>
</table>

Table 14.1 Principal and Interest Payments for a Treasury Inflation Protected Security
Indexed Bonds

- The first payment comes at the end of the first year, at $t = 1$. Because inflation over the year was 2%, the par value of the bond increases from $1,000 to $1,020; since the coupon rate is 4%, the coupon payment is 4% of this amount, or $40.80.
Indexed Bonds

- Notice that par value increases by the inflation rate, and because the coupon payments are 4% of par, they too increase in proportion to the general price level. Therefore, the cash flows paid by the bond are fixed in *real* terms. When the bond matures, the investor receives a final coupon payment of $42.44 plus the (price-level-indexed) repayment of principal, $1,061.11.
Indexed Bonds

- The *nominal* rate of return on the bond in the first year is

Nominal return

\[
= \frac{\text{Interest} + \text{Price Appreciation}}{\text{Initial Price}}
\]

\[
= \frac{\$40.80 + \$20}{\$1,000}
\]

\[
= 6.08\%
\]
Indexed Bonds

- The real rate of return is precisely the 4% real yield on the bond:

\[
\text{Real return} = \frac{1 + \text{Nominal return}}{1 + \text{Inflation}} - 1
\]

\[
= \frac{1 + 6.08\%}{1 + 2\%} - 1
\]

\[
= .04 \text{ or } 4\%
\]
Indexed Bonds

One can show in a similar manner that the rate of return in each of the 3 years is 4% as long as the real yield on the bond remains constant. If real yields do change, then there will be capital gains or losses on the bond. In early 2003, the real yield on TIPS bonds was about 2.8%.
Because a bond’s coupon and principal repayments all occur months or years in the future, the price an investor would be willing to pay for a claim to those payments depends on the value of dollars to be received in the future compared to dollars in hand today. This “present value” calculation depends in turn on market interest rates.
BOND PRICING

- As we saw in Chapter 5, the nominal risk-free interest rate equals the sum of (1) a real risk-free rate of return and (2) a premium above the real rate to compensate for expected inflation.
BOND PRICING

- In addition, because most bonds are not riskless, the discount rate will embody an additional premium that reflects bond-specific characteristics such as default risk, liquidity, tax attributes, call risk, and so on.
We simplify for now by assuming there is one interest rate that is appropriate for discounting cash flows of any maturity, but we can relax this assumption easily. In practice, there may be different discount rates for cash flows accruing in different periods. For the time being, however, we ignore this refinement.
To value a security, we discount its expected cash flows by the appropriate discount rate. The cash flows from a bond consist of coupon payments until the maturity date plus the final payment of par value. Therefore,

\[ \text{Bond value} = \text{Present value of coupons} + \text{Present value of par value} \]
If we call the maturity date $T$ and call the interest rate $r$, the bond value can be written as

$$\text{Bond value} = \sum_{t=1}^{T} \frac{\text{Coupon}}{(1 + r)^t} + \frac{\text{Par value}}{(1 + r)^T} \quad (14.1)$$
The summation sign in equation 14.1 directs us to add the present value of each coupon payment; each coupon is discounted based on the time until it will be paid. The first term on the right-hand side of equation 14.1 is the present value of an annuity. The second term is the present value of a single amount, the final payment of the bond’s par value.
We can rewrite equation 14.1 as follows:

$$\text{Price} = \text{Coupon} \times \frac{1}{r} \left(1 - \frac{1}{(1+r)^T}\right) + \text{Par value} \times \frac{1}{(1+r)^T}$$

$$= \text{Coupon} \times \text{Annuity factor}(r, T)$$

$$+ \text{Par value} \times \text{PV factor}(r, T)$$

(14.2)
EXAMPLE 14.2: Bond Pricing

- We discussed earlier an 8% coupon, 30-year maturity bond with par value of $1,000 paying 60 semiannual coupon payments of $40 each. Suppose that the interest rate is 8% annually, or \( r = 4\% \) per 6-month period.
EXAMPLE 14.2: Bond Pricing

- Then the value of the bond can be written as

\[
\text{Price} = \sum_{t=1}^{60} \frac{\$40}{(1 + 4\%)^t} + \frac{\$1,000}{(1 + 4\%)^{60}}
\]

\[
= \$40 \times \text{Annuity factor}(4\% , \ 60)
\]

\[
+ \$1,000 \times \text{PV factor}(4\% , \ 60)
\]

(14.3)
EXAMPLE 14.2: Bond Pricing

- It is easy to confirm that the present value of the bond’s 60 semiannual coupon payments of $40 each is $904.94 and that the $1,000 final payment of par value has a present value of $95.06, for a total bond value of $1,000. In this example, the coupon rate equals the market interest rate and the bond price equals par value.
EXAMPLE 14.2: Bond Pricing

- If the interest rate were not equal to the bond’s coupon rate, the bond would not sell at par value.
EXAMPLE 14.2: Bond Pricing

For example, if the interest rate were to rise to 10% (5% per 6 months), the bond’s price would fall by $189.29 to $810.71, as follows:

\[
\begin{align*}
$40 \times \text{Annuity factor}(5\%, 60) + $1,000 \times \text{PV factor}(5\%, 60) &= $757.17 + $53.54 \\
&= $810.71
\end{align*}
\]
BOND PRICING

- At a higher interest rate, the present value of the payments to be received by the bondholder is lower. Therefore, the bond price will fall as market interest rates rise. This illustrates a crucial general rule in bond valuation. When interest rates rise, bond prices must fall because the present value of the bond’s payments are obtained by discounting at a higher interest rate.
BOND PRICING

- Figure 14.3 shows the price of the 30-year, 8% coupon bond for a range of interest rates, including 8%, at which the bond sells at par, and 10%, at which it sells for $810.71. The negative slope illustrates the inverse relationship between prices and yields.
Figure 14.3
The inverse relationship between bond prices and yields. Price of an 8% coupon bond with 30-year maturity making semiannual payments
BOND PRICING

- Note also from the figure (and from Table 14.2) that the shape of the curve implies that an increase in the interest rate results in a price decline that is smaller than the price gain resulting from a decrease of equal magnitude in the interest rate. This property of bond prices is called convexity because of the convex shape of the bond price curve.
### Table 14.2  Bond Prices at Different Interest Rates
(8% coupon bond, coupons paid semiannually)

<table>
<thead>
<tr>
<th>Time to Maturity</th>
<th>Bond Price at Given Market Interest Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>4%</td>
</tr>
<tr>
<td>1 year</td>
<td>1,038.83</td>
</tr>
<tr>
<td>10 years</td>
<td>1,327.03</td>
</tr>
<tr>
<td>20 years</td>
<td>1,547.11</td>
</tr>
<tr>
<td>30 years</td>
<td>1,695.22</td>
</tr>
</tbody>
</table>
This curvature reflects the fact that progressive increase in the interest rate result in progressively smaller reductions in the bond price. Therefore, the price curve becomes flatter at higher interest rates.
Corporate bonds typically are issued at par value. This means that the underwriters of the bond issue (the firms that market the bonds to the public for the issuing corporation) must choose a coupon rate that very closely approximates market yields.
In a primary issue of bonds, the underwriters attempt to sell the newly issued bonds directly to their customers. If the coupon rate is inadequate, investors will not pay par value for the bonds.
BOND PRICING

- After the bonds are issued, bondholders may buy or sell bonds in secondary markets, such as the one operated by the New York Stock Exchange or the over-the-counter market, where most bonds trade. In these secondary markets, bond prices move in accordance with market forces. The bond prices fluctuate inversely with the market interest rate.
The inverse relationship between price and yield is a central feature of fixed-income securities. Interest rate fluctuations represent the main source of risk in the fixed-income market. This highlights one key factor that determines that sensitivity, namely, the maturity of the bond.
A general rule in evaluating bond price risk is that, keeping all other factors the same, the longer the maturity of the bond, the greater the sensitivity of price to fluctuations in the interest rate.
BOND PRICING

- For example, consider Table 14.2 which presents the price of an 8% coupon from 8% (the rate at which the bond sells at par value), the change in the bond price is smaller for shorter times to maturity.
This makes sense. If you buy the bond at par with an 8% coupon rate, and market rates subsequently rise, then you suffer a loss: You have tied up your money earning 8% when alternative investments offer higher returns. This is reflected in a capital loss on the bond—a fall in its market price.
The longer the period for which your money is tied up, the greater the loss, and correspondingly the greater the drop in the bond price.
In Table 14.2, the row for 1-year maturity bonds shows little price sensitivity—that is, with only 1 year’s earnings at stake, changes in interest rates are not too threatening. But for 30-year maturity bonds, interest rate swings have a large impact on bond prices.
This is why short-term Treasury securities such as T-bills are considered to be the safest. They are free not only of default risk but also largely of price risk attributable to interest rate volatility.

BOND PRICING
Equation 14.2 for bond prices assumes that the next coupon payment is in precisely one payment period, either a year for an annual payment bond or 6 months for a semiannual payment bond. But you probably want to be able to price bonds all 365 days of the year, not just on the one or two dates each year that it makes a coupon payment!
Bond Pricing between Coupon Dates

- We apply the same principles to pricing regardless of the date: we simply compute the present value of the remaining payments. However, if we are between coupon dates, there will be fractional periods remaining until each payment. Even if the principles are no more complicated, this certainly complicates the arithmetic computations.
Bond Pricing between Coupon Dates

- As we pointed out earlier, bond prices are typically quoted net of accrued interest. These prices, which appear in the financial press, are called flat prices. The actual invoice price that a buyer pays for the bond includes accrued interest. Thus,

Invoice price = Flat price + Accrued interest
Bond Pricing between Coupon Dates

- When a bond pays its coupon, flat price equals invoice price, since at that moment accrued interest reverts to zero. However, this will be the exceptional case, not the rule.
Bond Pricing between Coupon Dates

- The Excel pricing function provides the flat price of the bond. To find the invoice price, we need to add accrued interest. Fortunately, Excel also provides functions that count the days since the last coupon payment date and thus can be used to compute accrued interest.
BOND YIELDS

- We have noted that the current yield of a bond measures only the cash income provided by the bond as a percentage of bond price and ignores any prospective capital gains or losses.
We would like a measure of rate of return that accounts for both current income and the price increase or decrease over the bond’s life. The yield to maturity is the standard measure of the total rate of return. However, it is far from perfect, and we will explore several variations of this measure.
Yield to Maturity

- In practice, an investor considering the purchase of a bond is not quoted a promised rate of return. Instead, the investor must use the bond price, maturity date, and coupon payments to infer the return offered by the bond over its life.
Yield to Maturity

- The **yield to maturity (YTM)** is defined as the interest rate that makes the present value of a bond’s payments equal to its price. This interest rate is often viewed as a measure of the average rate of return that will be earned on a bond if it is bought now and held until maturity. To calculate the yield to maturity, we solve the bond price equation for the interest rate given the bond’s price.
EXAMPLE 14.3: Yield to Maturity

- Suppose an 8% coupon, 30-year bond is selling at $1,276.76. What average rate of return would be earned by an investor purchasing the bond at this price?
EXAMPLE 14.3: Yield to Maturity

- We find the interest rate at which the present value of the remaining 60 semiannual payments equals the bond price. This is the rate consistent with the observed price of the bond.
EXAMPLE 14.3: Yield to Maturity

Therefore, we solve for $r$ in the following equation:

$$1,276.76 = 40 \times \text{Annuity factor}(r, 60) + 1,000 \times \text{PV factor}(r, 60)$$
EXAMPLE 14.3: Yield to Maturity

- These equations have only one unknown variable, the interest rate, \( r \). You can use a financial calculator or spreadsheet to confirm that the solution is \( r = 0.30 \), or 3% per half-year. This is considered the bond’s yield to maturity.
EXAMPLE 14.3: Yield to Maturity

- The financial press reports yields on an annualized basis, and annualizes the bond’s semiannual yield using simple interest techniques, resulting in an annual percentage rate, or APR. Yields annualized using simple interest are also called “bond equivalent yields.”
EXAMPLE 14.3: Yield to Maturity

- Therefore, the semiannual yield would be doubled and reported in the newspaper as a bond equivalent yield of 6%. The effective annual yield of the bond, however, accounts for compound interest.
EXAMPLE 14.3: Yield to Maturity

- If one earns 3% interest every 6 months, then after 1 year, each dollar invested grows with interest to $1 \times (1.03)^2 = $1.0609, and the effective annual interest rate on the bond is 6.09%.
Yield to Maturity

- The bond’s yield to maturity is the internal rate of return on an investment in the bond. The yield to maturity can be interpreted as the compound rate of return over the life of the bond under the assumption that all bond coupons can be reinvested at that yield. Yield to maturity is widely accepted as a proxy for average return.
Yield to Maturity

- Yield to maturity is different from the current yield of a bond, which is the bond’s annual coupon payment divided by the bond price. For example, for the 8%, 30-year bond currently selling at $1,276.76, the current yield would be $80/$1,276.76 = .0627, or 6.27% per year.
Yield to Maturity

- In contrast, recall that the effective annual yield to maturity is 6.09%. For this bond, which is selling at a premium over par value ($1,276 rather than $1,000), the coupon payments by par value ($1,000) rather than by the bond price ($1,276).
Yield to Maturity

- In turn, the current yield exceeds yield to maturity because the yield to maturity accounts for the built-in capital loss on the bond; the bond bought today for $1,276 will eventually fall in value to $1,000 at maturity.
Yield to Maturity

- Example 14.3 illustrates a general rule: for **premium bonds** (bonds selling above par value), coupon rate is greater than current yield, which in turn is greater than yield to maturity. For **discount bonds** (bonds selling below par value), these relationships are reversed.
Yield to Maturity

- It is common to hear people talking loosely about the yield on a bond. In these cases, it is almost always the case that they are referring to the yield to maturity.
Yield to Call

- Yield to maturity is calculated on the assumption that the bond will be held until maturity. What if the bond is callable, however, and may be retired prior to the maturity date? How should we measure average rate of return for bonds subject to a call provision?
Yield to Call

- Figure 14.4 illustrates the risk of call to the bondholder.
Figure 14.4
Bond prices:
Callable and straight debt.
Coupon = 8%;
maturity = 30 years;
semiannual payments.
Yield to Call

- The colored line is the value at various market interest rates of a “straight” (i.e., noncallable) bond with par value $1,000, an 8% coupon rate, and a 30-year time to maturity. If interest rates fall, the bond price, which equals the present value of the promised payments, can rise substantially.
Yield to Call

- Now consider a bond that has the same coupon rate and maturity date but is callable at 110% of par value, or $1,100.
Yield to Call

- When interest rates fall, the present value of the bond’s *scheduled* payments rises, but the call provision allows the issuer to repurchase the bond at the call price. If the call price is less than the present value of the scheduled payments, the issuer can call the bond back from the bondholder.
Yield to Call

- The dark line in Figure 14.4 is the value of the callable bond. At high interest rates, the risk of call is negligible, and the values of the straight and callable bonds converge.
Yield to Call

- At lower rates, however, the values of the bonds begin to diverge, with the difference reflecting the value of the firm’s option to reclaim the callable bond at the call price. At very low rates, the bond is called, and its value is simply the call price, $1,100.
This analysis suggests that bond market analysts might be more interested in a bond’s yield to call rather than yield to maturity if the bond is especially vulnerable to being called.
Yield to Call

- The yield to call is calculated just like the yield to maturity except that the time until call replaces time until maturity, and the call price replaces the par value. This computation is sometimes called “yield to first call,” as it assumes the bond will be called as soon as the bond is first callable.
EXAMPLE 14.4: Yield to Call

- Suppose the 8% coupon, 30-year maturity bond sells for $1,150 and is callable in 10 years at a call price of $1,100. Its yield to maturity and yield to call would be calculated using the following inputs: (next slide)
<table>
<thead>
<tr>
<th></th>
<th>Yield to Call</th>
<th>Yield to Maturity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coupon payment</td>
<td>$40</td>
<td>$40</td>
</tr>
<tr>
<td>Number of semiannual periods</td>
<td>20 periods</td>
<td>60 periods</td>
</tr>
<tr>
<td>Final payment</td>
<td>$1,100</td>
<td>$1,000</td>
</tr>
<tr>
<td>Price</td>
<td>$1,150</td>
<td>$1,150</td>
</tr>
</tbody>
</table>
EXAMPLE 14.4: Yield to Call

- Yield to call is then 6.64%. [To confirm this on your calculator, input \( n = 20; PV = (-)1,150; FV = 1100; PMT = 40; \) compute \( i \) as 3.32%, or 6.64% bond equivalent yield].

Yield to maturity is 6.82%. [To confirm, input \( n = 60; PV = (-)1,150; FV = 1000; PMT = 40; \) compute \( i \) as 3.41% or 6.82% bond equivalent yield.]
Yield to Call

- We have noted that most callable bonds are issued with an initial period of call protection. In addition, an implicit form of call protection operates for bonds selling at deep discounts from their call prices. Even if interest rates fall a bit, deep-discount bonds still will sell below the call price and thus will not be subject to a call.
Yield to Call

- Premium bonds that might be selling near their call prices, however, are especially apt to be called if rates fall further. If interest rates fall, a callable premium bond is likely to provide a lower return than could be earned on a discount bond whose potential price appreciation is not limited by the likelihood of a call.
Yield to Call

- Investors in premium bonds often are more interested in the bond’s yield to call rather than yield to maturity as a consequence, because it may appear to them that the bond will be retired at the call date.
Realized Compound Yield versus Yield to Maturity

- We have noted that yield to maturity will equal the rate of return realized over the life of the bond if all coupons are reinvested at an interest rate equal to the bond’s yield to maturity.
Consider, for example, a 2-year bond selling at par value paying a 10% coupon once a year. The yield to maturity is 10%. If the $100 coupon payment is reinvested at an interest rate of 10%, the $1,000 investment in the bond will grow after 2 years to $1,210, as illustrated in Figure 14.5, panel A.
Figure 14.5
Growth of invested funds

A. Reinvestment Rate = 10%

Cash Flow: $100
Time: 0 1 2

Future Value:
$1,100

B. Reinvestment Rate = 8%

Cash Flow: $100
Time: 0 1 2

Future Value:
$1,100

\[
\begin{align*}
\text{Future Value} &= 100 \times 1.10 = \$1,100 \\
\text{Future Value} &= 100 \times 1.08 = \$1,080
\end{align*}
\]
Realized Compound Yield versus Yield to Maturity

- The coupon paid in the first year is reinvested and grows with interest to a second-year value of $110, which together with the second coupon payment and payment of par value in the second year results in a total value of $1,210.
Realized Compound Yield versus Yield to Maturity

- The compound growth rate of invested funds, therefore, is calculated from

\[ 1000(1 + y_{\text{realized}})^2 = 1,210 \]

\[ \Rightarrow y_{\text{realized}} = 0.10 = 10\% \]

With a reinvestment rate equal to the 10% yield to maturity, the realized compound yield equals yield to maturity.
But what if the reinvestment rate is not 10%? If the coupon can be invested at more than 10%, funds will grow to more than $1,210, and the realized compound return will exceed 10%. If the reinvestment rate is less than 10%, so will be the realized compound return. Consider the following example.
EXAMPLE 14.5: Realized Compound Yield

- If the interest rate earned on the first coupon is less than 10%, the final value of the investment will be less than $1,120, and the realized compound yield will be less than 10%. To illustrate, suppose the interest rate at which the coupon can be invested equals 8%.
EXAMPLE 14.5: Realized Compound Yield

- The following calculations are illustrated in Figure 14.5, panel B.

Future value of first coupon payment with earnings $100 \times 1.08 = \$108$

Cash payment in second year (final coupon plus par value) $\$1,100$

Total value of investment with reinvested coupons $\$1,208$
EXAMPLE 14.5: Realized Compound Yield

- The realized compound yield is computed by calculating the compound rate of growth of invested funds, assuming that all coupon payments are reinvested. The investor purchased the bond for par at $1,000, and this investment grew to $1,208.

\[ 1,000 \times (1 + y_{\text{realized}})^2 = 1,208 \]

\[ \Rightarrow y_{\text{realized}} = 0.0991 = 9.91\% \]
Realized Compound Yield versus Yield to Maturity

- Example 14.5 highlights the problem with conventional yield to maturity when reinvestment rates can change over time. Conventional yield to maturity will not equal realized compound return.
However, in an economy with future interest rate uncertainty, the rates at which interim coupons will be reinvested are not yet known. Therefore, although realized compound yield can be computed \textit{after} the investment period ends, it cannot be computed in advance without a forecast of future reinvestment rates. This reduces much of the attraction of the realized yield measure.
Realized Compound Yield versus Yield to Maturity

- Forecasting the realized compound yield over various holding periods or investment horizons is called horizon analysis. The forecast of total return depends on your forecasts of both the price of the bond when you sell it at the end of your horizon and the rate at which you are able to reinvest coupon income.
Realized Compound Yield versus Yield to Maturity

- The sales price depends in turn on the yield to maturity at the horizon date. With a longer investment horizon, however, reinvested coupons will be a larger component of your final proceeds.
EXAMPLE 14.6: Horizon Analysis

- Suppose you buy a 30-year, 7.5% (annual payment) coupon bond for $980 (when its yield to maturity is 7.67%) and plan to hold it for 20 years. Your forecast is that the bond’s yield to maturity will be 8% when it is sold and that the reinvestment rate on the coupons will be 6%.
EXAMPLE 14.6: Horizon Analysis

- At the end of your investment horizon, the bond will have 10 years remaining until expiration, so the forecast sales price (using a yield to maturity of 8%) will be $966.45. The 20 coupon payments will grow with compound interest to $2,758.92. (This is the future value of a 20-year $75 annuity with an interest rate of 6%.)
EXAMPLE 14.6: Horizon Analysis

- Based on these forecasts, your $980 investment will grow in 20 years to $966.45 + $2,758.92 = $3,725.37. This corresponds to an annualized compound return of 6.90%, calculated by solving for $r$ in the equation

$$980(1 + r)^{20} = 3,725.37.$$
As we noted earlier, a bond will sell at par value when its coupon rate equals the market interest rate. In these circumstances, the investor receives fair compensation for the time value of money in the form of the recurring interest payments. No further capital gain is necessary to provide fair compensation.
When the coupon rate is lower than the market interest rate, the coupon payments alone will not provide investors as high a return as they could earn elsewhere in the market.
To receive a fair return on such an investment, investors also need to earn price appreciation on their bonds. The bonds, therefore, would have to sell below par value to provide a “built-in” capital gain on the investment.
To illustrate this point, suppose a bond was issued several years ago when the interest rate was 7%. The bond’s annual coupon rate was thus set at 7%. (We will suppose for simplicity that the bond pays its coupon annually.) Now, with 3 years left in the bond’s life, the interest rate is 8% per year.
BOND PRICES OVER TIME

- The bond’s market price is the present value of the remaining annual coupons plus payment of par value. That present value is

$$70 \times \text{Annuity factor}(8\%, 3) + 1,000 \times \text{PV factor}(8\%, 3) = 974.23$$

which is less than par value.
In another year, after the next coupon is paid, the bond would sell at

\[ $70 \times \text{Annuity factor}(8\%, 2) + $1,000 \times \text{PV factor}(8\%, 2) = $982.17 \]

thereby yielding a capital gain over the year of $7.94.
If an investor had purchased the bond at $974.23, the total return over the year would equal the coupon payment plus capital gain, or $70 + $7.94 = $77.94. This represents a rate of return of $77.94/$974.23, or 8%, exactly the current rate of return available elsewhere in the market.
BOND PRICES OVER TIME

- When bond prices are set according to the present value formula, any discount from par value provides an anticipated capital gain that will augment a below-market coupon rate just sufficiently to provide a fair total rate of return.
Conversely, if the coupon rate exceeds the market interest rate, the interest income by itself is greater than that available elsewhere in the market. Investors will bid up the price of these bonds above their par values.
As the bonds approach maturity, they will fall in value because fewer of these above-market coupon payments remain. The resulting capital losses offset the large coupon payments so that the bondholder again receives only a fair rate of return.
**BOND PRICES OVER TIME**

- Figure 14.6 traces out the price paths of high- and low-coupon (net of accrued interest) as time to maturity approaches, at least for the case in which the market interest rate is constant. The low-coupon bond enjoys capital gains, whereas the high-coupon bond suffers capital losses.
Figure 14.6
Prices over time of 30-year maturity, 6.5% coupon bonds. Bond price approaches par value as maturity approaches.
We use these examples to show that each bond offers investors the same total rate of return. Although the capital gain versus income components differ, the price of each bond is set to provide competitive rates, as we should expect in well-functioning capital markets.
Security returns all should be comparable on an after-tax risk-adjusted basis. If they are not, investors will try to sell low-return securities, thereby driving down their prices until the total return at the new lower price is competitive with other securities.
Prices should continue to adjust until all securities are fairly priced in that expected returns are comparable, given appropriate risk and tax adjustments.
Yield to Maturity versus Holding-Period Return

- We just considered an example in which the holding-period return and the yield to maturity were equal: In our example, the bond yield started and ended the year at 8%, and the bond’s holding-period return also equaled 8%. This turns out to be a general result.
When the yield to maturity is unchanged over the period, the rate of return on the bond will equal that yield. As we noted, this should not be a surprising result: The bond must offer a rate of return competitive with those available on other securities.
However, when yields fluctuate, so will a bond’s rate of return. Unanticipated changes in market rates will result in unanticipated changes in bond returns, and after the fact, a bond’s holding-period return can be better or worse that the yield at which it initially sells.
Yield to Maturity versus Holding-Period Return

- An increase in the bond’s yield acts to reduce its price, which means that the holding-period return will be less than the initial yield. Conversely, a decline in yield will result in a holding-period return greater than the initial yield.
Consider a 30-year bond paying an annual coupon of $80 and selling at par value of $1,000. The bond’s initial yield to maturity is 8%. If the yield remains at 8% over the year, the bond price will remain at par, so the holding-period return also will be 8%. But if the yield falls below 8%, the bond price will increase.
Suppose the yield falls and the price increases to $1,050. Then the holding-period return is greater than 8%:

\[
\text{Holding-period return} = \frac{[$80 + ($1,050 - $1,000)]}{$1,000} = .13 \text{ or } 13\%
\]
Here is another way to think about the difference between yield to maturity and holding-period return.
Yield to Maturity versus Holding-Period Return

- Yield to maturity depends only on the bond’s coupon, current price, and par value at maturity. All of these values are observable today, so yield to maturity can be easily calculated. Yield to maturity can be interpreted as a measure of the average rate of return if the investment in the bond is held until the bond matures.
Yield to Maturity versus Holding-Period Return

- In contrast, holding-period return is the rate of return over a particular investment period and depends on the market price of the bond at the end of that holding period; of course this price is *not* known today. Since bond prices over the holding period will respond to unanticipated changes in interest rates, holding-period return can at most be forecast.
Zero-Coupon Bonds